COMPUTING RESERVES
OF MINERAL DEPOSITS: PRINCIPLES
AND CONVENTIONAL METHODS

By Constantine C. Popoff

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ABSTRACT

This report reviews and analyzes, by a simple analytical and logical reasoning, the conventional methods of reserve computations of mineral deposits described in various domestic and foreign publications. It brings together, formulates, and evaluates the principles underlying interpretation of exploration data; and ties such principles to the proposed classification of methods. The material is discussed in sufficient detail to allow general application.

INTRODUCTION

Computation of reserves is recognized by the mineral industry as a distinct operation of increasing importance in the evaluation of mineral deposits in all stages of their development. Previously, valuation was based on facts, experience, and intuition; methods have improved because our knowledge of mineral deposits, sampling, and mining techniques has increased.

Originally, computation methods followed practices of earth excavation and road construction, both standard surveying operations. Advances in earth sciences and engineering resulted in the modification of old and introduction of new methods.

The purpose of this investigation is to review some of the common methods and their modifications used in reserve computations of mineral deposits. The scope of this paper is limited to solid mineral deposits (that is, metal, non-metallic, coal, and oil shale), because the background data required, procedure, and methods for water, oil, and gas are dissimilar. An attempt is made to systematize and standardize the methods and terminology.

For convenience, the paper is divided into two parts. The first, "Principles", deals with assumptions and scientific principles underlying the
use of various methods, and provides a general discussion of the elements of computations, procedure, and errors of interpretation. The second part, "Conventional Methods", covers the following methods and their modifications: average factors and area (analogous and geologic blocks), mining blocks, cross sections (standard, linear, and isolines), triangular and polygonal prisms, and combinations of these.

The text often refers directly to ore deposits, because the problem of computing their reserves is generally more complicated due to diversity of form and size of mineral bodies and irregular distribution of values. The same methods are used for coal and nonmetallic deposits.

Statistical analysis is a valuable tool of research for all the methods of computations. Application of various methods of statistical analysis to sampling and exploration data is under continuing investigation by the Bureau of Mines. These methods and the use of computers for reserve computations are discussed by the Bureau and other scientists in several recent publications. They are beyond the scope and intent of this paper.

Reference is made to "exploration" workings for brevity; however, the text applies to mineral deposits in all stages of exploration, development, and exploitation. In the latter case, computations are particularly critical from the standpoint of economics but, once accepted, are usually subsidiary and routine operations.

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Grateful acknowledgment is made to J. A. Patterson, assistant chief of Ore reserve Branch, U.S. Atomic Energy Commission, Grand Junction, Colo., for the opportunity to study unpublished material on reserve computations.

PART 1. - PRINCIPLES

General

Significance of Computations

The purpose of reserve computations of a mineral body is to determine the quantity, the quality, and the amenability to commercial exploitation of raw material (ore, rock, coal, etc.). Computations are made during all stages of the life of a mining enterprise from discovery to robbing pillars and closing. They are the most responsible and irreplaceable tasks in the valuation of a mineral deposit. Efficiency in extraction and productiveness is impossible without accurate reserve computations.

Reserves are computed to determine the extent of exploration and development; distribution of values; annual output; probable and possible productive life of the mine; method of extraction and plant design; improvements in extraction, treatment, and processing; and requirements for capital, equipment, labor, power, and materials. Such computations are used to assist development planning; to determine production costs, efficiency of operations, and mining
losses; for quality control; for financing mining ventures; for sale, purchase, and consolidation of companies; to determine the production cost per unit of a marketable product; for accounting purposes such as depletion and depreciation; and in some States for tax purposes.

Requirements

No computations are justified unless called for and used; they should be made when required. The ideal method should be simple, rapid, reliable, consistent with the character of the mineral body and available data, and suitable for rapid checking. Computations are expected to be inexpensive when compared with the cost of exploration and development, and therefore, more complex methods are sometimes justifiable, particularly when labor-saving devices (calculators and computers) are available. In selecting a method, the peculiarities and conveniences of automation should be considered, as well as the magnitude and accuracy required.

The method should be selected carefully, procedures worked out in detail, and computations made accurately. Formulas should be simple. Properly selected procedures will facilitate the process of computations and provide the same degree of accuracy as more complicated methods.

Objective treatment of factual data is considered by many earth scientists the most important requirement. Harding, for example, states that his studies and formulas were provided by a desire to find "a method of calculating which eliminates all factors of test and judgment and rests on pure mathematics, ...a method which can be handled almost entirely by a calculating machine" (13).

Computations should also meet the purpose of the valuation and, when appropriate, illustrate the distribution of variables.

The reliability of reserve computations depends chiefly on the accuracy and completeness of our knowledge of the mineral deposit. It also depends on assumptions accepted for interpreting the variables, on boundaries of mineral bodies, on accuracy of averages, and on mathematical formulas. Requirements for the quantity and the density of observations for a certain category of resources depends primarily on the size and type of the mineral deposit.

During the last several decades the accuracy of computing reserves has gradually improved. This was made possible by outstanding advances in the field of economic geology; increased specialization; improvements in exploration, sampling, mining, and valuation; better interpretation of field information; use of statistical analysis; and more efficient management.

The growing use of data-processing machines has made it possible to record large amounts of exploration data in the form of punch cards, punched tape, magnetic disk, or magnetic tape. The computers permit application of

\footnote{Underlined numbers in parentheses refer to items in bibliography at the end of this report.}
two or more conventional methods and produce improved accuracy, increased speed, and labor and cost savings in reserve computations. The techniques and advantages of the use of computers are discussed in several recent publications (12, 21, 23-25, 38).

Criteria for Method Selection

In general, selecting a method for reserve computations depends upon the geology of the mineral deposit, exploration method, availability and reliability of factual data, purpose of computations, and the required degree of accuracy.

If computations are preliminary or are required immediately, simple methods, which do not demand construction of special maps, are selected. If computations are for mine design, the method selected depends on the contemplated mining system. The cutoff grade, recovery, dilution, efficiency of equipment and labor, and cost per unit of output vary with the system of extraction. A simple method may be adequate for open pit operations when selective extraction of waste or weakly mineralized rock is excluded. Computations of reserves for a bedded deposit is less complex than for high-grade, small volume, stock-type deposits with irregularly distributed values.

Exploration, whether random, by grid, or by cross-section lines, may also influence method selection. It is often desirable during exploration to use a method permitting step-by-step addition of reserves to previous figures instead of periodic recomputations.

The nature of the various methods should be carefully considered. Simple methods are preferred, but more complicated ones may be justified. Both extremes, oversimplification leading to complete disregard of the geologic nature of the deposit and overcomplication leading to unwarranted precision, expense, and even impracticability, should be avoided. The question of maximal use of all factual data collected in the process of exploration is an important consideration. Poor planning and overexploration results in excessive data not necessary for the accepted accuracy of computations.

Computing Reserves Procedure

Analysis of Exploration Data

Reserve computations of a mineral deposit is a technical task, consisting of several operations. The importance of following a definite procedure, properly selected for a certain deposit, cannot be overemphasized (41). The operations in order of their usual execution are geologic appraisal, exploration and sampling methods appraisal, exploration data appraisal, delineation of the mineral body, and selection of an appropriate method for computations.

The importance of the knowledge of the geology of the deposit for the understanding of the size, shape, and grade distribution, and for interpretation of exploration data has been emphasized by many scientists (29, 34). Geologic appraisal includes obtaining, checking, and presenting exploration
data in the form of graphs, tables, maps and sections of appropriate scale, and assuming a working hypothesis on the origin of mineralization. The exploration method; that is, the kind and density of workings and sampling, is studied to determine the adequacy and accuracy of the data from the standpoint of geology, geometric configuration of the mineral bodies, distribution pattern of variables, errors, and category of reserves. Such an appraisal is often supplemented by statistical analysis and by comparison with other deposits similar in type and form.

The analysis of exploration data, often the most neglected step in valuation, is accomplished by defining inside and outside parameters of economically minable portions of the mineral body; by determining the precision of measurements and analyses; and by determining whether the amount of exploration of various portions of the mineral body meets the requirements for computing reserves of a certain category.

Procedure

For reserve computations the mineral body is first delineated and then subdivided by several methods into segments or blocks of various degrees of reliability.

The usual procedure for volume computations is to substitute graphically the irregular shape of the mineral body by an imaginary and auxiliary one with base surface lying in the plane of a plan or longitudinal section; the other surface, irregular in form, shows distribution of thicknesses (fig. 1). This auxiliary body is then replaced by one or several simple solid figures, volumes of which can be computed by geometric formulas.

Division of the mineral body into blocks is done according to a selected method, so that each block can be directly related to one or a suite of factual exploration values.

The reserves of the entire body are computed by determining areas and volumes for each block, converting block volumes to tonnages of raw mineral material, determining average grades and tonnages of valuable components, and finally, tabulating the result of blocks of the same category and, if possible, assessing the reliability of computations.

Main Elements

Reserve computations require a knowledge of the dimensional and qualitative features of the mineral body. This knowledge is gained directly by observations (measurements, chemical analyses, and tests) and indirectly by assumptions, interpretations, and computations. All values of the basic block elements, thickness, length, breadth, weight factor, and grade, whether they are single observations or computed averages, may be presented on maps by numbers pinpointed for a definite location, or as a line with the numerical length plotted to scale.
The system selected for measuring linear distances, areas, volumes, and weights should be followed throughout. Units of measure and weight and conversion factors for English and metric systems are given in appendix A. When selection is possible, the metric system is preferable; it saves time and reduces the chance of error.

The formulas for all methods are based on computing solids with their bases constructed in the plane; vertical thickness is used for horizontal plan; horizontal thickness for vertical or longitudinal sections; and true thickness for incline longitudinal sections drawn in the plane of the dip of the mineral body. The relationships between the true, horizontal, and vertical thicknesses are:

\[ t_{tr} = t_h \sin \beta = t_v \cos \beta \]  

where \( \beta \) is the true dip of the body (fig. 2A).

In reserve computations, the true strike and the true dip of the deposit, or its portion under consideration, are determined first of all. Corrections for dip taken in a direction not perpendicular to the strike are made from specially prepared tables or by a protractor described in field geology textbooks (11, 23).

FIGURE 1. - Transforming a True Mineral Body Into an Imaginary Auxiliary One. A, Vertical section of true body; B, vertical section of distorted auxiliary body.
FIGURE 2. True, Horizontal and Vertical Thicknesses—Analytical Relationship. 

- A, $t_{tr} = t_h \sin \beta = t_v \cos \beta$; 
- B, strike correction - $t_{tr} = t_{ap} \cos \alpha$ ($\beta = 90^\circ \theta = 0$); 
- C, dip correction ($\alpha = 0^\circ$); 
- D, general case - block diagram.
In exploration, thicknesses usually are measured at oblique directions to the true strike and the true dip of the body. Such apparent thicknesses are corrected by graphical means, by trigonometric formulas, and by charts and tables.

When thickness is measured at an oblique angle to the strike, it is corrected by $\cos \alpha$ in a simple case of a vertical ore body and horizontal workings, where $\alpha$ is an angle between the apparent thickness plane and a plane perpendicular to the strike (fig. 2B).

When the thickness is measured at an oblique angle to the true dip of the deposit, the true $t_r$, horizontal $t_h$, and vertical $t_v$ thicknesses in a simple case of $\alpha$ equal 0° (fig. 2B) are

$$t_r = t_{ap} \sin (\beta + \theta),$$  \hspace{1cm} \text{(2)}

$$t_h = t_{ap} \frac{\sin (\beta + \theta)}{\sin \beta},$$  \hspace{1cm} \text{(3)}

and

$$t_v = t_{ap} \frac{\sin (\beta + \theta)}{\cos \beta},$$  \hspace{1cm} \text{(4)}

where $\theta$ is an angle intersecting the body in the plane of the true dip; $\beta$ - the dip of the body; and $t_{ap}$ - apparent thickness.

In a general case when the dip of the body and hole inclination are unconformable (hole crossing the body at sharp angle to the strike and to the dip), the thicknesses are found by formulas (fig. 2C)

$$t_r = t_{ap} \cos \beta \cos \theta \left( \cos \alpha \tan \beta + \tan \theta \right), \text{ or}$$

$$t_r = t_{ap} \left( \cos \alpha \sin \beta \cos \theta + \cos \beta \sin \theta \right),$$  \hspace{1cm} \text{(5)}

$$t_h = t_{ap} \left( \cos \alpha \cos \theta + \cotan \beta \sin \theta \right),$$  \hspace{1cm} \text{(6)}

and

$$t_v = t_{ap} \cos \theta \left( \cos \alpha \tan \beta + \tan \theta \right),$$  \hspace{1cm} \text{(7)}

where $\alpha$ is an angle between the plane of the dip and the plane of hole direction; $\beta$ - dip of body; and $\theta$ - angle of the hole intersecting the body.

Basic Assumptions

Whatever method is used for reserve computations several assumptions are taken for granted. The chief one is that the basic elements of a mineral body observed or established at any station (surface exposure, drill hole, or underground workings) change or extend to the adjoining area according to an appropriate principle of interpretation of exploration data. It is assumed also that observations are made in conformity with the nature of a given deposit.
and that the samples are taken with the same precision and are representative of a selected portion of the mineral body. When observations are doubtful or inadequate in number, the results of the computations are uncertain or erroneous.

Another important presumption is that the mineral deposit has been explored by an appropriate exploration method, and that the net of workings prove the continuity of the body. This hypothesis permits consideration of any element as having a constant value for a block, segment, or the entire mineral body. Thus, the problem of computation is reduced to determining the volume of a block, segment, or body by mathematical means.

Finally, it is assumed for the purpose of computations, that the true and often complex form of the mineral body can be represented with reasonable accuracy by a hypothetical body with a base surface lying in the plan or section. Such an idealized body may embrace the entire deposit, or it may be composed of large segments or small blocks, each characterized by a single or a suite of recorded variables.

**Principles of Interpretation**

The reasoning used in interpretation of variables between any two adjacent observations in a mineral body determines the block construction and the accuracy of computations. These principles are analytical, natural or intrinsic, and empirical. The analytical group includes the rule of gradual straight line changes of all basic elements of a mineral body and the rule of nearest points, or equal sphere of influence. Geologic, technologic, and economic criteria make up the natural or intrinsic group, and generalization the empirical rule of interpretation.

Application of the analytical and natural principles of interpretation are limited to specific conditions, necessary and sufficient for certain type and size of deposits, and for certain categories of resources.

**Rule of Gradual Changes**

**Mathematical Procedure**

According to the rule of gradual changes or law of linear function, all elements of a mineral body that can be expressed numerically change gradually and continuously along a straight line connecting two adjoining stations (fig. 3). Let us consider two adjoining stations or holes A and B with thicknesses...
t₁ and t₂. Location of a point C on line AB with a given thickness t₁ may be found analytically and graphically by this rule; vice versa, the thickness t₂ may be found for a given point C by similar procedures. To locate point C with given thickness t₁ on line AB, triangles A₁C₀C₁ and A₁B₂B₁ are similar, thus

\[
\frac{A_1 C_2}{A_1 B_2} = \frac{C_1 C_2}{B_1 B_2} = \frac{t_c - t_1}{t_2 - t_1},
\]

A₁C₂ = AC and A₁B₂ = AB,

\[
AC = \frac{(t_2 - t_1)}{(t_2 - t_1)} AB;
\]

and to determine thickness t₂ for a given point C,

\[
\frac{C_2 C_0}{B_2 B_0} = \frac{A_1 C_2}{A_1 B_2} = \frac{(t_2 - t_1)}{(t_2 - t_1)} AB, \\
t_2 - t_1 = \frac{AC}{AB} (t_2 - t_1),
\]

and

\[
t_c = \frac{AC}{AB} (t_2 - t_1) + t_1. 
\]

In surveying, equations (8) and (9) are known as formulas of simple interpretation.

The rule of gradual changes can be applied to other parameters of a mineral body such as grade and weight factors, as well as to areas, linear reserves, volumes, and tonnages. It may be used also in delineating the commercial portion of the deposit and to determine a given value at an unknown point on the extension of a line beyond known stations. In practice, interpolation and extrapolation are done by graphic means.

Graphic Procedure

To determine by vectors point D with a given thickness t₄ of 5 feet, on a line AB with t₁ equal to 4 feet at station A and t₂ equal to 7 feet at station B, raise a perpendicular from station B equal to t₂ - t₄, or 2 feet, and drop a perpendicular from station A equal to t₄ - t₁, or 1 foot (fig. 4). Connect points A₂ and B₂ with a straight line; the intersection of lines A₂B₂ and AB is point D.
FIGURE 5. - Finding Thickness $t_d$ for Point D (Rule of Gradual Changes).

$A_2 \rightarrow D_2 \rightarrow B_2$

$A \rightarrow D \rightarrow B$

$\ell_1 = 4 \text{ ft}$

$\ell_2 = 7 \text{ ft}$

$D_2 \rightarrow t_d$

FIGURE 6. - Finding Thickness $t_c$ for Point C and Finding Point D for Thickness $t_d = 5$ Feet by Means of a Special Template (Rule of Gradual Changes).

$A = 4 \text{ ft}$

$B = 7 \text{ ft}$

$t_c = 6.2 \text{ ft}$

To find thickness $t_c$ turn the template to have $A$ at $4 \text{ ft}$ and $B$ at $7 \text{ ft}$.
Finding thickness \( t_d \) for any given point on line AB is illustrated by figure 5. Raise a perpendicular \( A_A \) equal to \( t_1 \) from station A and a perpendicular \( B_B \) equal to \( t_2 \) from station B. Connect the points \( A_A \) and \( B_B \) with straight line. The thickness for a given point D on line AB will be the length of a perpendicular \( D_D \) raised from D to the line \( A_A B_B \).

For the same purpose special templates may be used, consisting of a series of parallel lines drawn to scale on tracing cloth or engraved on clear plastic. The lines are equidistant and may be marked by appropriate unit values (fig. 6). To find, in our example, thickness \( t_c \) for point C on line AB, the template is placed so that B will coincide with the line marked 7 feet. Let us turn the template around point B until A intersects the line marked 4. A supplementary line drawn parallel to other lines will show thickness for point C equal to 6.2 feet. A point D with thickness of 5 feet between the values of A and B is found by a simple intersection of 5.0 feet line and line AB.

Rule of Nearest Points, or Equal Influence

According to the rule of nearest points, or "equal sphere of influence", the value of any point between two stations is considered constant, equal to the value of the nearest station. In a general case of holes A and B with thickness \( t_1 \) and \( t_2 \), the value of each one extends to the midpoint X between holes (fig. 7). In practice the midpoint is found by the intersection of two arcs of a circle with radii slightly more than half the distance between the stations, or by a special template (fig. 8). Any point on line AB, except X, is inside the "linear influence" of a station A or B and nearer to it than to the adjoining one. Thus, this property gives the rule its name of nearest points. In the section AB the areas of influence for a given thickness \( t_1 \) of hole A and for a given thickness \( t_2 \) of hole B are shown by different patterns (fig. 7).

The rule of nearest points is widely used for construction of equal spheres of influence for areas and volumes of individual workings. The

![FIGURE 7. Interpretation of Values Between Two Adjoining Holes in Section (Rule of Nearest Points).]
application varies due to the type and distribution of workings, and whether these workings are presented on maps in the form of dots or lines. Several of the more common cases are described below.

**Case of Two Underground Intersecting Workings**

When a drift and a raise intersect in a plane of a map, the areas of equal influence of each working are found by bisecting the angle between them. Any point on the bisector is equidistant from both workings, and any point within each area of influence is nearer to the adjoining working than to the other (fig. 9).

**Case of Two Parallel Workings**

A line constructed equidistant between two parallel workings, such as trenches, drifts, crosscuts, raises, and drill holes, will divide the intervening area into two areas of equal influence, each satisfying the property of nearest points (fig. 10).

**General Case of Underground Workings**

When a block is developed by drifts and raises on all four sides, the area between them is divided into four areas of influence by a combination of angle bisectors and parallel lines (fig. 11). To satisfy the property of
equal influence for each pair of workings in the block, only the illustrated construction is possible.

**Case of Equilateral Triangle**

In an equilateral triangle the areas of influence of each vertex are found by constructing perpendicular bisectors from the midpoint of each side (fig. 12A). The intersection of the bisectors is equidistant from the vertices; it is the center of a circle passing through the three vertexes.

By constructing angle bisectors, different shaped areas are formed (fig. 12B) in comparison with perpendicular bisectors. This manner of dividing a triangle is called the rule of gravity.

**Case of an Obtuse Triangle**

In an obtuse triangle, the angle bisectors will divide the figure into three areas different in shape, but equal in size (fig. 13). The point of
intersection of the angle bisectors, or the center of the gravity of the triangle, is much closer to the vertex of the obtuse angle than to the other vertexes. Therefore, the center as well as other points of the triangle, are inconsistent with the rule of nearest points.

Areas of influence constructed by perpendicular bisectors in an obtuse triangle are of different sizes, but the vertexes are equidistant from the intersection of the perpendiculars. The areas of influence of such a construction satisfy the property of the rule of nearest points.

**General Case**

Both manners of construction of areas of influence are used in computing reserves. The angle bisector manner is limited to workings presented on plans and sections as lines, such as intersecting underground workings (fig. 11).

**FIGURE 12.** Construction of Areas of Influence in Equilateral Triangles (Rule of Nearest Points). A, Perpendicular bisector manner of constructing areas of influence (areas are to vertexes A, B, and C); B, angle bisector manner of constructing areas of influence (areas are to lines AB, BC, and AC).

**FIGURE 13.** Perpendicular Bisector Versus Angle Bisector Manner of Constructing Areas of Influence in Obtuse Triangles. A, Perpendicular bisector manner of constructing areas of influence for vertexes A, B, and C; B, angle bisector manner of constructing areas of influence for lines AB, BC, and AC; C, angle bisector manner of constructing areas of influence for vertexes A, B, and C. The last method is incorrect from the standpoint of rule of nearest points (O is closer to B than to A and C).
When the workings are presented on the map as dots (drill holes), the areas of influence of each one are found by the perpendicular manner of construction (fig. 14). In the latter case the angle bisector manner of construction is incorrect (fig. 15). For the quadrilateral figure (ABCD), the angle bisector construction may produce two different results, depending on how the triangles are drawn (fig. 16). Construction of areas by perpendicular bisectors (fig. 17) produces only one solution. For further discussion see section entitled "Method of Polygons".

An area of influence for the outside perimeter of the mineral body, or for an isolated hole, can be constructed by the rule of nearest points, when

Two solutions—depending on construction of triangles (both incorrect).

FIGURE 14. - Correct Construction of Areas of Influence (Polygons) by Perpendicular Bisectors (Rule of Nearest Points).

FIGURE 15. - Incorrect Construction of Polygons by Angle Bisectors (Rule of Gravity).

FIGURE 16. - Areas of Influence for Quadrilateral Figures (Rule of Gravity).
FIGURE 17. - Areas of Influence for Quadrilateral Figures (Rule of Nearest Points).

Only one solution (correct)

FIGURE 18. - Areas of Influence for a Square Block (Rule of Nearest Points).

a "standard" mean radius of influence for a certain category of reserves and type of deposit is accepted. Such areas may be constructed by a circle equal to the standard radius of influence (figs. 18 and 19).

Geologic and Mining Inference

When interpreting variables between two adjoining workings, the construction of segments or blocks of a mineral body may be governed by direct geologic, mining, or economic considerations. In a simple case of two drill holes with corresponding thicknesses $t_1$ and $t_2$ of ore and a prominent vertical fault between them, the sphere of influence (areas of influences for section and plan) may be assigned on basis of geologic interpretation, as illustrated in figure 20; that is, the thickness $t_1$ is consigned to ore between the hole A and the fault, and the thickness $t_2$ between the fault and hole B.
FIGURE 19. - Area of Influence for an Isolated Hole.

FIGURE 20. - Geologic Interpretation of Areas of Influence Between Two Adjoining Stations.
Motives for geologic inference include natural geologic boundaries due to structural features (synclines, anticlines, faults, or other dislocations, changes in strike or dip); changes in character of mineralization; thinning out or pitching of oreshoots; zoning; weathering; different physical properties; heterogenous composition; varied alteration; and presence of detrimental constituents, such as ash and sulfur in coal.

Common technologic, physiographic, and economic grounds for inference in construction of blocks are topography, thickness of overburden, ratio of overburden to thickness of mineral body, depth, water level, mining methods, processing methods, and cost of extraction; also property, section, township, and state boundaries. An example of block construction on the basis of structural changes in a phosphate rock deposit and availability of ore for open pit mining is given in figure 21.

![Diagram of geologic blocks](image-url)

**FIGURE 21.** Construction of Geologic Blocks on the Basis of Structural Changes.
Rule of Generalization

The rule of generalization is also known as the empirical method and, in its extreme, as the rule of thumb. It is used frequently for interpretation of exploration data. In contrast with the more objective interpretations described previously, such a rule is used rather arbitrarily. It is often adapted for lack of other criteria on the basis of limited experience, or as a matter of judgment and generally reflects past experience and opinion.

In many cases, the use of the rule is justifiable and unavoidable. Adapting a definite weight factor for reserve computations from other similar mineral deposits is probably the most common example. Selecting specific limits for the size of blocks in classifying reserves by categories for certain mineral deposits or assuming factor values for reserves on the basis of production data, rather than directly from widely spaced drill holes with irregular or doubtful values, are generalizations. Projecting continuity of a mineral body beyond the outlying workings along the strike or at depth and fixing cutoff boundaries for computations are other examples.

The following procedure was used for extrapolation of boundaries of uranium deposits between drill holes, some of which crossed ore, strongly mineralized ground, weakly mineralized ground, and barren rock. The cutoff boundary between two holes was selected as three-fourths or two-thirds the distance from ore to a strongly mineralized hole, one-half to a weakly mineralized hole, and one-third to a barren hole (63).

Many earth scientists exercise the above principle by arbitrarily reducing areas, average thicknesses, and grades for individual blocks and bodies and even correcting the computed reserves by subjective correction factors.

Variability Within Mineral Deposits

The preceding rules and the inferences based on geologic and mining conditions lead to well-defined methods for reserve computation. Commonly, however, the information available is such that inferences and the applicability of the preceding rules is not always clear cut. This may occur in the earlier stages of exploration where the amount of information is sparse. It may also occur in any stage of development if the natural variability within the mineral deposit is relatively high. This variability tends to mask the significance and relationships that are present in the information gathered on a mineral deposit. It is the function of statistical analysis to remove this mask and to assess the significance and relationships that are inherent in a set of data. If the variability within a deposit is relatively high, the application of some statistical procedure may be necessary before an intelligent selection of method of reserve computation can be made. On the other hand, if the variability is relatively small as compared to meaningful patterns and trends in the information, the selection of a method of reserve computation may require no prior statistical treatment of the data.

In the mining industry statistical procedures have been used in examining mine and exploration data to detect patterns and trends, to correlate
variables, and "to develop numerical data from which the reliability of estimates can be assessed" (32). Partly because these procedures are well adapted to electronic computing, they have been useful in obtaining the maximum amount of information from sparse exploration data and in handling large amounts of data from operating properties. The application of statistical methods to the results of sampling and reserve computations has been discussed in many publications (4-5, 15-18, 22, 62).

Statistical procedures are generally useful in isolating changes or variability that is due to chance from changes that are "real." The reliability of estimates is developed from the variability that is due to chance. In data obtained from mineral deposits, this variability may arise from a random distribution of values within the region considered. As the region considered is enlarged, "real" changes or trends invariably appear and the values within this enlarged region can no longer be considered random. It is generally useful and logical to consider changes in values within a deposit as the superposition of changes which are real or due to a trend and changes which are due to chance and explained by a locally random distribution of values.

According to most earth scientists, an assumption of random distribution of variables is contrary to the basic geologic hypothesis of the origin of mineral deposits, particularly sedimentary (53). This school considers each deposit a geochemical field, a structural field, or combination of both. Commercial concentration and distribution of valuable components within such fields result from the genesis of the deposit. The natural processes governing deposition and migration of minerals may be superimposed upon each other, or even be adverse to each other, thus creating an intricate distribution of valuable components. Advocates of the geochemical school consider grade and thickness changes in a mineral body to be due to the mode of origin and hidden irregularities in their distribution. The variables and their reliability depend on the place of observation in the mineral body. Thus, according to this school, each variable is a function of space of coordinates XYZ. Grade at a given point may deviate from mean grade, but the degree of deviation depends on the morphology of the body and on the particulars of observations and sampling.

On the other hand, the opponents of the above hypothesis believe that adverse geologic processes together with local and accidental changes, usually produce no clear orderly regularities in the thickness and grade of the mineral body. Much of this difference of opinion might be explained by the scale on which the phenomena is viewed.

Weighting

Weighting is the operation of assigning factors to each of a number of observations to represent their relative value, allocation, or importance when compared with other observations of the same suite. In the mining industry, the principle of weighting is widely used in computing averages of variables and reserves of mineral bodies. Allocations of weights are made in units of length, area, volume, and tonnage on the basis of different principles of interpretation, mainly the rule of nearest points, geologic, mining, and other considerations.
The use of weighting in each particular case depends on the analysis of exploratory data. In sectional sampling, across a wide mineral body, weighting may be compulsory for computation of average grade over the entire width of a vein with different metal values near the hanging wall and footwall. In all methods of reserve computations, the principle of weighting is applied to individual blocks of different sizes to determine average thickness and average grade of the entire deposit.

In certain cases, weighting by an area of influence is not appropriate. For a region within which the values are randomly distributed, no sample by definition has an area of influence; hence, weighting samples within this region by an area of influence is not logical in obtaining an average for the region. Thus, the rule of nearest points is not applicable for this case. However, this does not mean the samples from such a region should not be weighted for some other reason when computing the region average.

Application

All the above principles of interpretation are used in valuation of mineral deposits. A study of the common methods of computations discloses that block construction is usually based on one definite principle, and other principles, often secondary in importance, applied as supplementary operations (table 1). The principles of statistical analysis, weighting, and generalization are used in all conventional methods.

**TABLE 1. - Principles of interpretation of exploration data used in construction of blocks and reserve computations**

(XXX - Predominant; XX - Supplementary but influential; X - Secondary)

<table>
<thead>
<tr>
<th>Reserve computations: Conventional methods and modifications</th>
<th>Intrinsic</th>
<th>Analytical</th>
<th>Rule of generalization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Geologic</td>
<td>Mining and economics</td>
<td>Rule of gradual changes</td>
</tr>
<tr>
<td>Average factors and area:</td>
<td>XXX</td>
<td>XX</td>
<td>-</td>
</tr>
<tr>
<td>Analogous</td>
<td>XXX</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Geologic blocks</td>
<td>XXX</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Mining blocks</td>
<td>XX</td>
<td>XXX</td>
<td>-</td>
</tr>
<tr>
<td>Cross sections:</td>
<td>XX</td>
<td>XX</td>
<td>XXX</td>
</tr>
<tr>
<td>Standard</td>
<td>XX</td>
<td>XX</td>
<td>XXX</td>
</tr>
<tr>
<td>Linear</td>
<td>XX</td>
<td>XX</td>
<td>XXX</td>
</tr>
<tr>
<td>Isolines</td>
<td>XX</td>
<td>XX</td>
<td>XXX</td>
</tr>
<tr>
<td>Triangular prisms</td>
<td>X</td>
<td>-</td>
<td>XXX</td>
</tr>
<tr>
<td>Polygonal prisms</td>
<td>X</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The leading principle in average factors and area methods is based on geologic criteria. Mining, economic, and to lesser extent, geologic criteria
support the mining blocks method. The rule of gradual changes is basic to the method of triangular prisms and the rule of nearest points to the method of polygonal prisms. The rule of gradual changes is the predominant principle in the standard and isolines cross-section methods, and the rule of nearest points is used in the linear cross-section method.

Computations

Basic Parameters

The basic parameters for computing reserves of a mineral deposit include **thickness and area** - quantitative indicators of form, size, and volume of the mineral body; **grade** - the qualitative indicator of values and their distribution in the deposit; and **weight factor or specific gravity** - indicator for tonnage computations.

In most deposits thickness and grade vary from place to place in greater degree than the weight factor. For simplicity the latter is considered constant in this report.

Thickness and Area

Measurements of the thickness of a mineral body are taken directly by a series of observations, scaled from maps and sections, or computed, and then arithmetically averaged,

\[
t_{\text{avg}} = \frac{t_1 + t_2 + t_3 + \ldots + t_n}{n}.
\]  

Area is measured directly from maps by planimetering, by the use of specially constructed templates, by geometric computations, and indirectly by computing from survey data.

Planimetering

At least two planimeter readings taken in opposite directions are necessary to achieve correct results. If these readings vary by less than 2 percent, the average is accepted as true. The scale of the selected maps should meet the accuracy requirements of the smallest area measured.

Templates

Templates may be of square pattern, where each square has a certain unit-area value; of dotted pattern, where each dot is the center of a unit of equal area; or, of parallel lines pattern with a series of equidistant lines drawn to scale (figs. 22, 23, 24). The use of templates with the first two patterns is self-evident. In the case of the parallel lines template the lengths of all lines within the mineral body are totaled; the sum of lengths multiplied by the unit value of the scale equal the total area. Two different positions of a template are taken for precise measurements and the average accepted as
FIGURE 22. - Square Pattern Template.

One dot = 10 sq units

S = 460 sq units

FIGURE 23. - Dotted Pattern Template.

FIGURE 24. - Parallel Lines Template.
the true area. In practice, the square pattern is used when the area is 50 units or less.

*Geometric Computations*

Irregularly shaped areas may be divided into simple geometric figures; that is, triangles, squares, tetragons, and trapezoids. The dimensions of each figure can be scaled from maps or deduced from survey notes and the area computed. The total area is equal to the sum of the calculated figures. The most common formulas for plain figures, triangle, square, rectangle, and parallelogram are well known. Formulas for the trapezoid follow.

**Trapezoid Formula.** - An area of a simple trapezoid is

\[ S = \frac{(a + b)}{2} h, \]

where \(a\) and \(b\) are parallel sides of the figure and \(h\) the perpendicular distance (fig. 25).

**Trapezoidal Rule.** - An irregular area may be subdivided into an even number of trapezoidal figures by a series of equidistant parallel lines, or ordinates (fig. 26). Assuming that the boundaries of the strips between the ordinates are straight lines, the entire irregular area may be computed by the trapezoidal rule,
where \( h \) is a common interval between parallel lines or ordinates, and \( a_1, a_2, \ldots, a_n \) are the lengths of each ordinate.

It is obvious that the greater the number of strips the greater is the precision of the formula.

**Simpson's Rule.** - The computation of an irregular area by Simpson's rule (fig. 27) is based on the assumption that the curved boundaries of each strip are parabolas passing through consecutive points. If the number of offsets are odd and the number of strips even, the irregular area is computed by Simpson's formula (42, v. 2, p. 36-13),

\[
S = \frac{1}{3} h \left( a_1 + 2 \sum a_{odd} + 4 \sum a_{even} + a_n \right), \tag{13}
\]

where \( \sum a_{odd} \) - the sum of odd offsets

\( \sum a_{even} \) - the sum of even offsets.

If the number of offsets is even (and number of strips odd), one of the end-area trapezoids is computed separately and added to the results computed by the formula. Other, and lesser known, trapezoidal formulas for determining area are Durand's and Weddle's rules described in engineering handbooks.

\[
S = \frac{1}{3} h \left( a_1 + \frac{a_2 + a_3}{2} + \frac{a_4 + a_5}{2} + \ldots + \frac{a_{n-1} + a_n}{2} \right)
\]

or

\[
S = h \left[ \frac{(a_1 + a_n)}{2} + a_2 + a_3 + \ldots + a_{n-1} \right]. \tag{12}
\]
Indirect Methods

Some areas may be computed from survey notes by double meridian distances or by the coordinate method, described in civil engineering handbooks.

Volume

The volume of a block is computed from direct or indirect measurements of length (L), breadth (B), and thickness (T) by the parallelepiped formula,

\[ V = LTB. \]  \hspace{2cm} (14)

In practice mineral bodies are irregular, and it is necessary to substitute true volume by an equivolume body of solid geometric configuration for the use of simple formulas for volume computations.

When the area S is directly measured on the map and the average thickness \( t_a \) is computed mathematically, the general formula for the deposit is

\[ V = St_a. \]  \hspace{2cm} (15)

If a mineral body is subdivided into segments or blocks for computations, the volume of the entire body will be

\[ V = V_1 + V_2 + V_3 + \ldots + V_n = t_1 S_1 + t_2 S_2 + t_3 S_3 + \ldots + t_n S_n, \] \hspace{2cm} (16)

where \( S_1, S_2, S_3, \ldots, S_n \) are block areas and

\( t_1, t_2, t_3, \ldots, t_n \) are average thicknesses of individual blocks.

Various methods of block construction are discussed in part 2 of this report. It is obvious that the substitution error of the true volume of a mineral body with auxiliary blocks depends on the knowledge of the form and size of the body. In addition, the accuracy of computations depends on the number of blocks, variations in the size of blocks, and precision obtained by the formulas.

Weight

Tonnage Factors

Conversion of volume to tonnage of raw mineral material (ore, rock, and coal) varies, depending on the system of measures used. Common formulas used in computing tonnages are

\[ Q = \frac{V}{F} \] \hspace{2cm} and \hspace{2cm} \[ Q = Vf. \] \hspace{2cm} (17)

In the first formula \( F \) is the volume-tonnage factor and is usually expressed in cubic feet per ton. In the second formula \( f \) is the tonnage-volume factor and is usually expressed in weight-units per cubic foot.
Both weight factors are interrelated and are determined on the basis of past production, experimental mining, or adapted from similar deposits. They also may be determined by measuring excavations, or by special laboratory tests. Techniques of these determinations are described in several publications (36, 40, 42). In some cases tonnage factors may be computed from the mineral composition after corrections for porosity and moisture content of raw material are made.

In many deposits the weight factors vary substantially owing to the mineral and grade composition. The relationship between weight and grade often may be expressed graphically; thus, weight factors can be determined for appropriate grade of each individual block.

The ore tonnage of the entire body is determined by formula,

\[ Q = Q_1 + Q_2 + Q_3 + \ldots + Q_n = V_1 f_1 + V_2 f_2 + V_3 f_3 + \ldots + V_n f_n. \] (18)

**Specific Gravity**

Conversion of volume to tonnage (metric) is made by the formula,

\[ Q = VD, \] (19)

where \( D \) is specific gravity or density of raw mineral material.

The specific gravity of the mineral matter can be determined by direct tests of dried and crushed samples. Specific gravity of rock in place, or "rock specific gravity-natural" may be expressed by

\[ D_{sat} = \frac{D_m (1 - P_o)}{(1 - M_o)}, \] (20)

where \( D_m \) is the specific gravity of the mineral matter determined by tests of crushed and dried rock.

\( P_o \) is porosity in percent pore space to unit of volume.

\( M_o \) is moisture in percent weight loss upon drying. Mead offers a convenient diagram for the English system of weights and measures, showing the influence of porosity, moisture, and specific gravity on the tonnage factor (36).

Specific gravity may be calculated theoretically as an average of the specific gravity of all of the minerals in the deposit, or according to the weighted average percent of each mineral in the rock. For convenience, the calculated deviations of specific gravity for various grades may be presented graphically.

**Conversion Formulas**

The volume-tonnage factor, \( F \), is computed from specific gravity by the following formulas (42, v. 2, p. 25-20):
For short ton \( F_s \). t. = \( \frac{2,000}{62.5} \) \( \text{ft}^3/\text{s.t.} \), and

For long ton \( F_l \). t. = \( \frac{2,240}{62.5} \) \( \text{ft}^3/\text{L.t.} \),

where 62.5 lb is the weight of 1 cubic foot of water at 4° C.

The tonnage-volume factor, \( f \), is computed by

\[
f = \frac{\text{Volume in cubic feet}}{\text{Density in lb/ft}^3} = \frac{2,000}{62.5} \text{ or } \frac{2,240}{62.5} \text{ or } 62.5 \text{ D.}
\]

Volume in cubic feet can be converted to cubic meters by multiplying by 0.028 or dividing by 35.3. Metric tons are converted to short tons by multiplying 1.102 or to long tons by 0.984 (Appendix A).

When the density of a particular body varies appreciably from one place to another owing to the relative amounts of minerals with wide ranges in specific gravities, such as galena and iron oxide versus quartz, more accurate results are obtained by

\[
Q = V_1 D_1 + V_2 D_2 + V_3 D_3 + \ldots + V_n D_n \quad (23)
\]

where \( D_1, D_2, \ldots, D_n \) are specific gravities of separate blocks \( V_1, V_2, \ldots, V_n \).

Grade

Grade computation of a mineral body is a critical and important operation that can be done by various formulas:

1. Simple arithmetic mean (unweighted).
2. Weighted per width or thickness.
3. Weighted per width and length, or area.
4. Weighted by frequency of occurrence.
5. Weighted by the square of the frequency.
6. Weighted by frequency and assay (35).

The problems of sampling of various types of mineral deposits and methods of statistical analysis used in evaluating exploration data, and in computing average grade of workings, blocks, and commercial portions of bodies, are beyond the scope of this paper.

Generally, average grade of a mineral body is computed using conventional methods of reserve computations; the formulas used are
Type of problem | Assumption | Equation | Equation No.
--- | --- | --- | ---
Arithmetic average. | All blocks are equal in area, thickness, and weight factor. | \( c_{av} = \frac{c_1 + c_2 + c_3 + \ldots + c_n}{n} \) | (24)
Thickness-weighted average. | All blocks are equal in area and have the same weight factor. | \( c_{av} = \frac{c_1 t_1 + c_2 t_2 + c_3 t_3 + \ldots + c_n t_n}{t_1 + t_2 + t_3 + \ldots + t_n} \) | (25)
Area-weighted average. | All blocks have constant thickness and weight factor, but different areas. | \( c_{av} = \frac{c_1 S_1 + c_2 S_2 + c_3 S_3 + \ldots + c_n S_n}{S_1 + S_2 + S_3 + \ldots + S_n} \) | (26)
Volumetric average (volume-weighted average). | Weight factors of all blocks are the same. | \( c_{av} = \frac{c_1 V_1 + c_2 V_2 + c_3 V_3 + \ldots + c_n V_n}{V_1 + V_2 + V_3 + \ldots + V_n} \) | (27)
Gravimetric average (tonnage-weighted average). | Tonnages and grades of blocks are different. | \( c_{av} = \frac{c_1 Q_1 + c_2 Q_2 + c_3 Q_3 + \ldots + c_n Q_n}{Q_1 + Q_2 + Q_3 + \ldots + Q_n} \) | (28)

*Q₁, Q₂, Q₃, ..., Qₙ* are reserves of raw mineral material in individual blocks, in tons.

Reserves of valuable components are determined by formula

\[ P = Q c_{av} \]  
(29a)

where P is the sum of reserves of each valuable component of individual blocks P₁, P₂, P₃, ..., Pₙ and Q is the sum of reserves of raw material Q₁, Q₂, Q₃, ..., Qₙ. The average grade of deposit is determined by formula

\[ c_{av} = \frac{P}{Q} \]  
(29b)

**Errors**

Accuracy Versus Precision

The terms "accuracy" and "precision" as related to reserve computations are defined in this report as follows: The variance between a single observation or a computed average of an element of a mineral body and its true value indicates the degree of accuracy or exactness of such observations or estimates. An accurate measurement is free from all errors. Precision indicates only the degree of fluctuation in a certain suite of variables with respect to their proximity to each other.

The distinction between accuracy and precision is well illustrated graphically in the following example for a suite of chemical analyses (1, 57).

1. Accurate and precise (fig. 28A).
2. Inaccurate but precise (figs. 28B and 28C).
3. Accurate but not precise (fig. 28D).
4. Inaccurate and not precise (figs. 28E and 28F).

The errors in reserve computations may be divided into three groups: errors of interpretation (often labeled geologic), technical, and analytical.
Errors of Interpretation or Analogy

Errors of interpretation often called errors of analogy, representation, details, and geology are due to the accepted hypothesis of the origin of the deposit, assumption of geologic similarity to other deposits, interpretation or assumption of the uniform changes of the basic elements, and the continuity of the body along the strike and at depth. They are errors of judgment and, consequently, depend on the training and experience of the person appraising or conducting the investigation.

The results of exploration are generally disclosed by a series of plans and sections representing the mineral body in graphic form. Thus, the exactness of our knowledge of a definite mineral deposit depends on the correctness of the maps, which in turn, depends on the type of mineral deposit, kind and density of workings, and precision of all measurements and qualitative assays and tests.

Technical

Technical errors are those due to imperfections in instruments and techniques used in determining all variables. Errors, random, biased, or both, should be corrected to prevent downgrading or upgrading of individual observations, since erroneous variables influence interpretation of boundaries and
computation of basic parameters of the mineral body, and consequently, of the size and value of reserves.

Oversight errors due to faulty copying or typing and recording of samples are excluded from this discussion.

Random

Random, casual, or accidental technical errors are "those whose causes are unknown and undeterminate" (54, p. 454). These errors are mostly erratic, and may be of great magnitude (figs. 28D, 28E, 28F); they fluctuate on both sides of the true value and, when the average of a sufficient number of variables is computed, compensate each other.

Permissible random errors used in reserve computations of variables are given in table 2 (31, 46, 57, 59, 66), and for chemical analyses of various elements and grades in table 3 (57).

To prevent random errors in chemical analysis, some laboratories process two portions of the same sample, regular and duplicate. Both determinations are then compared with permissible deviation values (such as listed in table 3) and, if within range, the final result is reported as an average.

To find random errors for a suite of chemical analyses, periodic (monthly or quarterly) control analyses are made. These repeated samples are given in code in the amount of 3 to 5 percent of the total number of samples but not less than 30 (67). The magnitude of random error between control and regular analyses should not exceed the permissible limits adopted by the laboratory.

Biased

Biased or systematic errors are "those which affect all measurements alike" (54, p. 454). They are due to imperfections of instrument, equipment, and accepted techniques of observations. For chemical analyses such errors may be due to inexperienced personnel, inferior quality reagents, and selection of an improper method for a given sample. Most likely the errors are in one direction; that is, are persistently either more or less than the true value. Such errors affect the mean values because they are not compensating.

The presence and the magnitude of biased errors may be disclosed by special studies. To determine biased errors in chemical analyses, for example (67), control analyses of the same samples are made in a reputable laboratory by similar methods and procedures. Such outside laboratory control analyses are made for large projects at least twice a year; the number of analyses should be 3 to 5 percent of the total with a minimum of not less than 30 samples. If a substantial error is found and proved further by a third party, a correction factor should then be applied to all samples analyzed in the first laboratory.
### TABLE 2. Technical errors in determining basic parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Original observations</th>
<th>Precision, plus-minus</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thickness</strong></td>
<td>Tape (for 1 meter)</td>
<td>1.0 percent</td>
<td>Precision increases with ore thickness and decreases with irregularities of ore body boundaries.</td>
</tr>
<tr>
<td></td>
<td>From plans</td>
<td>Above 1.5 percent</td>
<td>Depends on scale and drawings.</td>
</tr>
<tr>
<td><strong>Length</strong></td>
<td>Surveying</td>
<td>0.5 percent and less</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Drawing</td>
<td>Up to 2.0 percent</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Brunton</td>
<td>0.5 to 2.0 degrees</td>
<td>Depending on scale</td>
</tr>
<tr>
<td><strong>Angle</strong></td>
<td>Surveying data</td>
<td>0.5 percent and less</td>
<td>Depending on angle of measurement, exposure, and convenience.</td>
</tr>
<tr>
<td></td>
<td>Planimeter (100-400 cm²)</td>
<td>1.5 to 0.3 percent (area)</td>
<td>Precision increases with the increase of area size.</td>
</tr>
<tr>
<td></td>
<td>Template</td>
<td>Up to 3.0 percent</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Maps:</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Scale 1: 200</td>
<td>0.5 percent</td>
<td>Depends on method of determination, type of ore, etc.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25 percent</td>
<td>Do.</td>
</tr>
<tr>
<td></td>
<td>Scale 1:5000</td>
<td>3 to 10 percent</td>
<td>Do.</td>
</tr>
<tr>
<td><strong>Specific gravity or weight factor</strong></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td><strong>Grade</strong></td>
<td>Sample taking</td>
<td>Varied</td>
<td>Depends on sampling method, sample processing, chemical analyses, type of deposit, method of computation, commodity, etc.</td>
</tr>
<tr>
<td></td>
<td>Sample processing</td>
<td>1.0 percent</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Chemical analysis</td>
<td>1 to 20 percent and more</td>
<td>Do.</td>
</tr>
</tbody>
</table>

Technical errors in connection with reserve computations of an ore body (range, accuracy, plus-minus):

- Surface and underground workings: 2.0 to 3.0 percent for thick and up to 10 percent for thin bodies—depending on the irregularities of thicknesses and grade.
- Drill holes: several to 30 percent and more—depending on exploration technique, core recovery, and type of deposits.
- Maps: 1.0 percent and above depending on scale.
- Sample taking: varied depending on scale.
- Do.
- Do. depending on scale.
- Do.
- For uniform ore: 3 to 20 percent depending on method of determination, porosity, moisture content, fissuring, etc.
- For complex composition ores: 20 percent and more.
- See table 3.
TABLE 3. - Permissible average for random technical errors
in chemical analyses
(The All-Union Committee on Mineral Reserves, U.S.S.R.)

<table>
<thead>
<tr>
<th>Component and grade range of raw material, percent, except as noted</th>
<th>Permissible average error in percent to the grade determined (plus-minus)</th>
<th>Component and grade range of raw material, percent, except as noted</th>
<th>Permissible average error in percent to the grade determined (plus-minus)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum oxide (Al₂O₃):</td>
<td></td>
<td>Copper:</td>
<td></td>
</tr>
<tr>
<td>Above 20</td>
<td>2 - 4</td>
<td>Above 3</td>
<td>3 - 7</td>
</tr>
<tr>
<td>5 - 20</td>
<td>4 - 8</td>
<td>0.5 - 3</td>
<td>7 - 10</td>
</tr>
<tr>
<td>1 - 5</td>
<td>8 - 20</td>
<td>Below 0.5</td>
<td>10 - 15</td>
</tr>
<tr>
<td>Antimony:</td>
<td></td>
<td>Gold, g/m ton (1 g = 15.432 grains):</td>
<td></td>
</tr>
<tr>
<td>Above 2</td>
<td>3 - 12</td>
<td>Below 0.1 mm:¹</td>
<td></td>
</tr>
<tr>
<td>0.2 - 2</td>
<td>12 - 20</td>
<td>Above 64</td>
<td>2.5</td>
</tr>
<tr>
<td>Arsenic:</td>
<td></td>
<td>16 - 64</td>
<td>2.5</td>
</tr>
<tr>
<td>Above 2</td>
<td>1 - 5</td>
<td>4 - 16</td>
<td>5 - 10</td>
</tr>
<tr>
<td>0.5 - 2</td>
<td>5 - 7</td>
<td>Below 4</td>
<td>15</td>
</tr>
<tr>
<td>Below 0.5</td>
<td>10</td>
<td>Below 0.6 mm:²</td>
<td></td>
</tr>
<tr>
<td>Barium sulfate (BaSO₄):</td>
<td></td>
<td>Above 64</td>
<td></td>
</tr>
<tr>
<td>Above 5</td>
<td>1 - 7</td>
<td>16 - 64</td>
<td></td>
</tr>
<tr>
<td>1 - 5</td>
<td>7 - 15</td>
<td>4 - 16</td>
<td></td>
</tr>
<tr>
<td>Beryllium:</td>
<td></td>
<td>Below 4</td>
<td></td>
</tr>
<tr>
<td>5 - 10</td>
<td>3 - 5</td>
<td>Above 0.6 mm:³</td>
<td></td>
</tr>
<tr>
<td>0.1 - 5</td>
<td>5 - 10</td>
<td>Above 64</td>
<td>7</td>
</tr>
<tr>
<td>0.01 - 0.1</td>
<td>10 - 30</td>
<td>16 - 64</td>
<td>7 - 15</td>
</tr>
<tr>
<td>Below 0.01</td>
<td>30</td>
<td>4 - 16</td>
<td></td>
</tr>
<tr>
<td>Bismuth:</td>
<td></td>
<td>Below 4</td>
<td></td>
</tr>
<tr>
<td>Above 0.6</td>
<td>5 - 15</td>
<td>Above 64</td>
<td></td>
</tr>
<tr>
<td>0.2 - 0.6</td>
<td>15 - 20</td>
<td>16 - 64</td>
<td></td>
</tr>
<tr>
<td>Cadmium:</td>
<td></td>
<td>4 - 16</td>
<td></td>
</tr>
<tr>
<td>Above 1</td>
<td>3 - 5</td>
<td>Below 4</td>
<td></td>
</tr>
<tr>
<td>0.1 - 1</td>
<td>5 - 10</td>
<td>Above 0.6 mm:³</td>
<td></td>
</tr>
<tr>
<td>0.01 - 0.1</td>
<td>10 - 30</td>
<td>Above 64</td>
<td>7</td>
</tr>
<tr>
<td>Below 0.01</td>
<td>30</td>
<td>16 - 64</td>
<td></td>
</tr>
<tr>
<td>Calcium oxide (CaO):</td>
<td></td>
<td>Iron, g/m ton (1 g = 15.432 grains):</td>
<td></td>
</tr>
<tr>
<td>Above 25</td>
<td>3 - 5</td>
<td>Above 30</td>
<td>1 - 2</td>
</tr>
<tr>
<td>5 - 25</td>
<td>5 - 10</td>
<td>10 - 30</td>
<td>2 - 4</td>
</tr>
<tr>
<td>1 - 5</td>
<td>10 - 25</td>
<td>5 - 10</td>
<td>4 - 8</td>
</tr>
<tr>
<td>Chromium:</td>
<td></td>
<td>Iron oxide (FeO):</td>
<td></td>
</tr>
<tr>
<td>Above 10</td>
<td>1 - 3</td>
<td>Above 5</td>
<td>2 - 4</td>
</tr>
<tr>
<td>1 - 10</td>
<td>3 - 7</td>
<td>1 - 5</td>
<td>4 - 7</td>
</tr>
<tr>
<td>Below 1</td>
<td>7</td>
<td>Below 0.5</td>
<td>12</td>
</tr>
<tr>
<td>Cobalt:</td>
<td></td>
<td>Magnesium oxide (MgO):</td>
<td></td>
</tr>
<tr>
<td>Above 0.5</td>
<td>2 - 6</td>
<td>Above 5</td>
<td>3 - 10</td>
</tr>
<tr>
<td>Below 0.5</td>
<td>6</td>
<td>1 - 5</td>
<td>10 - 20</td>
</tr>
<tr>
<td>Columbium:</td>
<td></td>
<td>Manganese:</td>
<td></td>
</tr>
<tr>
<td>Above 10</td>
<td>3 - 5</td>
<td>Above 5</td>
<td>2 - 4</td>
</tr>
<tr>
<td>1 - 10</td>
<td>5 - 10</td>
<td>1 - 5</td>
<td>4 - 7</td>
</tr>
<tr>
<td>0.1 - 1</td>
<td>10 - 20</td>
<td>0.05 - 1</td>
<td>7 - 20</td>
</tr>
<tr>
<td>Below 0.1</td>
<td>20</td>
<td>Mercury:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Above 2</td>
<td>4 - 7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25 - 2</td>
<td>7 - 15</td>
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<tr>
<td></td>
<td></td>
<td>Below 0.25</td>
<td>15 - 20</td>
</tr>
</tbody>
</table>

See footnotes at end of table.
TABLE 3. - Permissible average for random technical errors in chemical analyses--Continued

(The All-Union Committee on Mineral Reserves, U.S.S.R.)

<table>
<thead>
<tr>
<th>Component and grade range of raw material, percent, except as noted</th>
<th>Permissible average error in percent to the grade determined (plus-minus)</th>
<th>Component and grade range of raw material, percent, except as noted</th>
<th>Permissible average error in percent to the grade determined (plus-minus)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molybdenum:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 1 .</td>
<td>2 - 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25 - 1 .</td>
<td>5 - 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below 0.25 .</td>
<td>10 - 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nickel:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 5 .</td>
<td>3 - 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 - 1 .</td>
<td>7 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below 0.2 .</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phosphorus:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 3 .</td>
<td>3 - 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03 - 0.3 .</td>
<td>7 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silicon dioxide (SiO₂):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 - 50 .</td>
<td>2 - 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 - 30 .</td>
<td>3 - 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 - 10 .</td>
<td>8 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silver, grains per ton:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 100 .</td>
<td>1 - 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 - 100 .</td>
<td>3 - 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 - 30 .</td>
<td>5 - 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sulfur:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 20 .</td>
<td>1 - 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 20 .</td>
<td>2 - 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05 - 1 .</td>
<td>5 - 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tantalum:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 10 .</td>
<td>3 - 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 10 .</td>
<td>5 - 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 - 1 .</td>
<td>10 - 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below 0.1 .</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tin:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 1 .</td>
<td>3 - 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25 - 1 .</td>
<td>7 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05 - 0.25 .</td>
<td>15 - 30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Titanium dioxide (Ti₃O₇):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 - 15 .</td>
<td>2 - 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 - 2 .</td>
<td>5 - 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tungsten trioxide (WO₃):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 1 .</td>
<td>3 - 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25 - 1 .</td>
<td>8 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05 - 0.25 .</td>
<td>15 - 30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vanadium:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 0.5 .</td>
<td>3 - 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.06 - 0.5 .</td>
<td>10 - 30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zinc:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 25 .</td>
<td>2 - 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 - 25 .</td>
<td>3 - 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 - 10 .</td>
<td>6 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below 0.5 .</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zirconium:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above 3 .</td>
<td>2 - 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 3 .</td>
<td>5 - 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 - 1 .</td>
<td>10 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below 0.1 .</td>
<td>15 - 25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1Samples with finely dispersed gold; mainly in sulfides.
2Samples with average grain size gold; in sulfides and quartz.
3Samples with large grain size, often visible, gold; mainly in quartz.

Source: Reference (57), table 7, pp. 67-68.

The correction factor may be computed as a ratio of the average grade of the control to the average grade of the regular analyses; that is

\[
E = \frac{C_c}{C_r}.
\]

The factor E is applied to regular samples to receive the correct results. For example, a regular suite of copper samples averaged 0.80 percent. Control samples averaged 1.0 percent copper. The correction factor is
The control analyses are 25 percent higher than the regular.

Analytical

Some analytical errors of reserve computations will be discussed in part 2. In general, the accuracy of computations increases with the number of blocks dividing the mineral body, provided the same accuracy is maintained in construction of each block. The error of a separate block may be high, but for a group of blocks representing the entire body the relative errors are balanced according to the law of averages;

blocks are equal in tonnage,

\[ M_{av} = \frac{M_1 + M_2 + M_3 + \ldots + M_n}{N}; \]

and

blocks are unequal in tonnage of valuable constituent

\[ M_{av} = \frac{M_1 P_1 + M_2 P_2 + \ldots + M_n P_n}{NP}, \]

where \( M_{av} \) is average relative error of mineral body, and \( M_1, M_2, M_3, \ldots, M_n \) are relative errors of individual blocks (in percent).

PART 2. - CONVENTIONAL METHODS

General Features and Classification

For reserve computations the mineral deposit, reduced and distorted by mapping, is converted to an analogous geometric body composed of one, several, or an aggregate of close-order solids, that best express the size, shape, and distribution of the variables. Construction of these blocks depends on the method selected. Some methods offer two or more manners of block construction, thus introducing subjectivity. In such a case a certain manner of construction is accepted as appropriate, preferably on the basis of geology, mining, and economics.

Numerous methods of reserve computations are described in the literature; some are only slight modifications of the most common ones. Depending on the criteria used in substituting the explored bodies by auxiliary blocks and on the manner of computing averages for variables, the conventional methods may be classified into four groups.

Group 1, average factors and area methods, embraces analogous and geologic blocks methods. Areas are delineated by geologic and, in part, by mining and economic criteria, and the basic elements (thickness, grade, and weight factors) are determined directly, computed, or taken from other portions of the same or similar deposits.
Group 2, **mining blocks method**, involves delineation of block areas by underground workings and by geologic and economic considerations; the factors for each block are computed in various ways. As the name implies the method is used mainly for extraction.

Group 3, **cross-section methods**, includes standard, linear, and isolines. The mineral body is delineated and the blocks are constructed on the basis of certain principles of interpretation of exploration data; the parameters of blocks and the entire body are determined in various ways.

Group 4, **analytical methods**, divides the mineral body graphically into blocks of simple geometric forms—triangular or polygonal prisms. The factors for each block are determined directly, computed as an arithmetic average, or in other ways.

Special studies of the usage of various methods were made in the U.S.S.R. Thousands of mineral deposits were explored and reserves computed and approved by the All-Union Committee on Mineral Reserves. The results for metal, non-metal, and coal and oil-shale deposits for the years 1941-61 and for solid mineral deposits for years 1941-47, 1951, and 1954 are given in appendix B (table B-1) (57). The predominant methods were--

<table>
<thead>
<tr>
<th>Methods, percent</th>
<th>Average factors and area</th>
<th>Cross sections</th>
<th>Polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal and oil shale deposits.......</td>
<td>69</td>
<td>-</td>
<td>30</td>
</tr>
<tr>
<td>Nonmetallic deposits.............</td>
<td>46</td>
<td>37</td>
<td>14</td>
</tr>
<tr>
<td>Ore deposits.....................</td>
<td>37</td>
<td>48</td>
<td>14*</td>
</tr>
</tbody>
</table>

*Including mining blocks method.

It is also reported in the U.S.S.R. that in computing reserves the use of average factors and area and cross section methods together had increased from 30 (1941-47) to a total of 82 percent (1954) of all projects recorded (appendix B, table B-2).

A comparison of the use of various methods by 44 metal mines, described by Jackson and Knaebel in "Sampling and Estimation of Ore Deposits" (28) published in 1932, shows that the mining blocks and cross-section methods were predominant in the mining industry (table 4).

**TABLE 4. Usage of various methods for reserve computations for metal mines in U.S. (1932)**

<table>
<thead>
<tr>
<th>Methods:</th>
<th>Percent</th>
<th>Methods:</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average factors and area.......</td>
<td>20</td>
<td>Polygons..........................</td>
<td>4</td>
</tr>
<tr>
<td>Mining blocks....................</td>
<td>42</td>
<td>Triangles.......................</td>
<td>2</td>
</tr>
<tr>
<td>Cross sections...................</td>
<td>32</td>
<td>Total...........................</td>
<td>100</td>
</tr>
</tbody>
</table>

1For 44 active mines described by Charles F. Jackson and John B. Knaebel. Sampling and Estimation of Ore Deposits, BuMines Bull. 356, 1932, pp. 125-249.
Average Factors and Area Methods

Assumptions and Characteristics Features

Average factors and area methods of reserve computations have been variously described as arithmetic average, weighted average, average depth and area, statistical, analogous (by analogy), geologic blocks, and general outline (27-28, 46, 57, 63). In this report these methods are discussed under the titles of analogous and geologic blocks.

Average factors and area methods are all based on the assumption that certain segments or blocks of the mineral body being considered are similar in geology and technology to sections previously studied, or to blocks or even bodies that have been explored or mined out. For computations the body is divided into segments or blocks constructed on the basis of geology, mining, and economic; that is, structure, thickness, grade, value, depth, and overburden. In some cases, the qualitative characteristics found in one part of the body may be accepted, for the purpose of computations, as representative of the block or the entire mineral body.

If the blocks are of equal size each observation and sample analysis has an equal influence in determining average factors. If the number of variables in a block are in sufficient quantity, average factors may be computed and studied by statistical analysis; on the other hand, the method of analogy may be used where only one observation is available. A number of segments or blocks with different controlling factors requires the use of the method of geologic blocks.

Formulas for computations of reserves range from simple to complex equations. Aside from the usual variables of grade, thickness, and density, more complex factors such as, tons recovered per unit of area, volume, or weight may be used.

Method of Analogy

Analogy is the inference that certain admitted partial resemblances probably imply further similarity. The method emphasizes qualitative similarity of the geology of a given block to an analogous and better known block of the same or similar body.

Variables for computations may be taken from a single or a number of observations, or computed from data gathered from the same or similar deposits. Such variables may be accepted as constant factors for other parts of the same body, other deposits, or even districts. When the geology of a given area or deposit is considered analogous to another area or deposit, a single observation may be adequate for reserve computations of a certain commodity. Reserves computed may belong to any category. The method is widely used in extraction operations when others are difficult to apply. In reserve computations of mica in pegmatites, for example, production records may be considered sufficient and accurate for assigning mica grade to the unmined portion of the vein below and between mined-out blocks.
Average Factors

The arithmetic average is the simplest variation of the analogous method. No auxiliary figures are constructed; thickness and grade are determined by a simple average of available data (fig. 29 and table 5). Grade may be determined also by thickness-weighting of individual grade observations from ore intersecting workings in the mineral body and even from adjoining parallel bodies and by extending the results to the unexplored block or to the entire mineral body (table 6). In case of numerous samples (observations) the average grade may be determined by statistical analysis (15-17). Reserve computations and the determination of the block-weighted average factors for the entire body is illustrated by table 7.

The formulas used are average thickness (formula 10), average grade (formula 24), thickness-weighted average grade (formula 25), volume of mineral body (formula 15), tonnage of raw material (formula 18), and tonnage of valuable component (formula 29a).

### FIGURE 29.
- Arithmetic Average Method of Computing Thickness. (For recapitulation of reserves for mineral body and determination of average grade, see table 5.)
TABLE 5. - Determination of average thickness and average grade for a block by arithmetic average procedure

<table>
<thead>
<tr>
<th>Workings (n), numbers</th>
<th>Thickness (t), ft</th>
<th>Grade (c), percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( t_1 )</td>
<td>( c_1 )</td>
</tr>
<tr>
<td>2.</td>
<td>( t_2 )</td>
<td>( c_2 )</td>
</tr>
<tr>
<td>3.</td>
<td>( t_3 )</td>
<td>( c_3 )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N.</td>
<td>( t_n )</td>
<td>( c_n )</td>
</tr>
<tr>
<td>Total</td>
<td>( \sum_{i=1}^{n} t_i )</td>
<td>( \sum_{i=1}^{n} c_i )</td>
</tr>
<tr>
<td>Average</td>
<td>( t_{av} = \frac{\sum_{i=1}^{n} t_i}{n} )</td>
<td>( c_{av} = \frac{\sum_{i=1}^{n} c_i}{n} )</td>
</tr>
</tbody>
</table>

TABLE 6. - Determination of thickness-weighted average grade for a block

<table>
<thead>
<tr>
<th>Workings (n), numbers</th>
<th>Thickness (t), ft</th>
<th>Grade (c), percent</th>
<th>Product, ( t \times c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( t_1 )</td>
<td>( c_1 )</td>
<td>( t_1 c_1 )</td>
</tr>
<tr>
<td>2.</td>
<td>( t_2 )</td>
<td>( c_2 )</td>
<td>( t_2 c_2 )</td>
</tr>
<tr>
<td>3.</td>
<td>( t_3 )</td>
<td>( c_3 )</td>
<td>( t_3 c_3 )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N.</td>
<td>( t_n )</td>
<td>( c_n )</td>
<td>( t_n c_n )</td>
</tr>
<tr>
<td>Average (block A).....</td>
<td>( \sum_{i=1}^{n} t_i )</td>
<td>( \sum_{i=1}^{n} c_i )</td>
<td>( \sum_{i=1}^{n} t_i c_i )</td>
</tr>
</tbody>
</table>

Note: Thickness-weighted average grade \( (c_{av}) \), percent: \( c_{av} = \frac{\sum_{i=1}^{n} t_i c_i}{\sum_{i=1}^{n} t_i} \).

The arithmetic average procedure is the simplest and most rapid method of computation; accuracy depends on the quality, quantity, density, and distribution of observations; in turn these factors depend on the genetic type and size of the deposit.

The system is accurate in uniform deposits; accuracy decreases in nonuniform deposits, even if the distribution of observations is done by a regular pattern but with insufficient density. In complex deposits accuracy is greater in regularly distributed workings than in irregular. An important disadvantage of this system is the lack of quality and quantity distribution of valuable components in space.
### TABLE 7. - Computation of reserves and average factors for the entire body

<table>
<thead>
<tr>
<th>Block</th>
<th>Area (S), sq ft</th>
<th>Thickness (t), ft</th>
<th>Volume (V), cu ft</th>
<th>Weight factor (F), cu ft/ton</th>
<th>Raw material reserves (Q), tons</th>
<th>Valuable component</th>
<th>Average grade (c), percent</th>
<th>Reserves (P), tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>A......</td>
<td>S_a</td>
<td>t_a</td>
<td>V_a</td>
<td>F</td>
<td>Q_a</td>
<td>c_a</td>
<td>P_a</td>
<td></td>
</tr>
<tr>
<td>B......</td>
<td>S_b</td>
<td>t_b</td>
<td>V_b</td>
<td>F</td>
<td>Q_b</td>
<td>c_b</td>
<td>P_b</td>
<td></td>
</tr>
<tr>
<td>N......</td>
<td>S_n</td>
<td>t_n</td>
<td>V_n</td>
<td>F</td>
<td>Q_n</td>
<td>c_n</td>
<td>P_n</td>
<td></td>
</tr>
<tr>
<td>Total...</td>
<td>n ∑ S</td>
<td>n ∑ V</td>
<td>n ∑ Q</td>
<td>n ∑ P</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average...</td>
<td>n ∑ V/ n = i=1</td>
<td>n ∑ S/ n = i=1</td>
<td>n ∑ Q/ n = i=1</td>
<td>n ∑ P/ n = i=1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Statistical Factors

Factors for reserve computations of a certain mineral commodity are determined in terms of production or value yield on the basis of exploration, past mining experience, or smelter returns gathered for the same mineral body, or even similar ones. These factors are usually expressed as percent of component or value recovered per unit of area, volume, or weight; that is

<table>
<thead>
<tr>
<th>Raw material reserves (Q), tons</th>
<th>Valuable component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ounces per cubic yard or ton (short, long); cents per cubic yard, square foot or square yard; milligrams per cubic yard or ton.</td>
<td>Grams per cubic meter or metric ton.</td>
</tr>
<tr>
<td>Pounds per cubic yard or ton.</td>
<td>Kilograms per cubic meter or metric ton.</td>
</tr>
<tr>
<td>Tons per square foot, acre or section.</td>
<td>Metric tons per square meter or square kilometer.</td>
</tr>
<tr>
<td>Percent of weight or pounds per ton.</td>
<td>Percent of weight or kilograms per ton.</td>
</tr>
</tbody>
</table>

The use of statistical factors often may be the only practical way of computing potential resources for a mine or district. The accuracy depends on the geologic interpretation of the mineral deposit, as well as on computed factors.
Reserves of uniform bedded deposits, coal, phosphate rock, and clay have been computed by the method of analogy from the results of spot drill holes and exposures in trenches and other surface workings. Reserves of phosphate rock available for open pit mining in Idaho were computed by the Bureau of Mines on the basis of detailed geologic sections, sample analyses, geologic maps, and other published data. Inferred reserves of a syncline, for example, illustrated in figure 21 in part I of this report were based on sections measured about 1 mile from the area (table 8).

TABLE 8. Reserve computations—method of analogy

<table>
<thead>
<tr>
<th>Block</th>
<th>Middle section (s), sq ft</th>
<th>Thickness accepted (t), ft</th>
<th>Volume (V), cu ft</th>
<th>Weight factor (F), cu ft/ton</th>
<th>Phosphate rock (Q), short tons</th>
<th>Grade (c), percent</th>
<th>P₂O₅, short tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block A, fig. 21, overturned section, 4,260 feet long</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acid grade</td>
<td>715,000</td>
<td>5.0</td>
<td>3,575,000</td>
<td>11.3</td>
<td>316,400</td>
<td>34.2</td>
<td>108,200</td>
</tr>
<tr>
<td>Do.</td>
<td>715,000</td>
<td>9.9</td>
<td>7,078,000</td>
<td>11.3</td>
<td>626,400</td>
<td>32.5</td>
<td>203,600</td>
</tr>
<tr>
<td>Furnace grade</td>
<td>715,000</td>
<td>6.5</td>
<td>4,668,000</td>
<td>12.0</td>
<td>387,300</td>
<td>26.9</td>
<td>104,200</td>
</tr>
<tr>
<td>Do.</td>
<td>715,000</td>
<td>15.0</td>
<td>10,725,000</td>
<td>12.0</td>
<td>893,700</td>
<td>28.1</td>
<td>251,100</td>
</tr>
<tr>
<td>Beneficiation grade</td>
<td>715,000</td>
<td>13.4</td>
<td>9,581,000</td>
<td>12.0</td>
<td>737,000</td>
<td>19.2</td>
<td>141,500</td>
</tr>
<tr>
<td>Total or average</td>
<td>715,000</td>
<td>49.8</td>
<td>35,607,000</td>
<td>-</td>
<td>2,960,800</td>
<td>27.3</td>
<td>808,600</td>
</tr>
</tbody>
</table>

Block B, fig. 21, normal dip, 2,050 feet long

| Acid grade          | 570,000                  | 5.0                       | 2,850,000         | 11.3                          | 252,200                        | 34.2               | 86,300           |
| Do.                  | 570,000                  | 9.9                       | 5,643,000         | 11.3                          | 499,400                        | 32.5               | 162,300          |
| Furnace grade       | 570,000                  | 6.5                       | 3,705,000         | 12.0                          | 308,800                        | 26.9               | 83,100           |
| Do.                  | 570,000                  | 15.0                      | 8,550,000         | 12.0                          | 712,500                        | 28.1               | 200,200          |
| Beneficiation grade | 570,000                  | 13.4                      | 7,638,000         | 13.0                          | 587,500                        | 19.2               | 112,800          |
| Total or average    | 570,000                  | 49.8                      | 28,386,000        | -                             | 2,360,400                      | 27.3               | 644,700          |

1 Computed for potential surface resources in a phosphorite deposit in Idaho. Section is composed of 2 acid grade zones (+31 percent P₂O₅), 2 furnace grade seams (24 to 31 percent P₂O₅), and the remainder beneficiation grade (18 to 24 percent P₂O₅). True bed thickness is 49.8 feet.

Method of Geologic Blocks

Although the method of geologic blocks has been widely used by earth scientists for many years, it was not until 1950 that its principles were first discussed and its name accepted (57); the procedure is also known as the method of analogy and general outline (63).

A geologic block may be the entire mineral deposit or a relatively small portion of it, outlined on a map by interpretation of exploratory data. The block sides may coincide with the natural boundaries of the deposit, or be delineated on the basis of geological features, structural deformations, or variations in thickness and grade. In addition, blocks also may be outlined on the basis of physiographic factors; adaptability to certain mining methods; availability of mineral raw material at depth; possibilities of utilization; requirements for beneficiation and processing; or property, section, township, or State boundaries.

The factors are determined from available exploratory data or may be adapted from results of spot sampling, production averaging, or data from other parts of the same deposit. Cutoff grade is determined by geologic and mining considerations and processing. Interpretation of data may be by the rules of gradual changes, nearest points, or generalization. The parameters of each geologic block and the entire
body are determined by procedure described for the method of analogy. The average grade of an individual block is computed either by the arithmetic average (table 5), weighted-average (table 6), or by statistical analysis. Reserves of each block are computed as the product of area and average factors; total reserves are the sum of all individual blocks (table 7).

Depending on the extent of the geologic knowledge of the deposit, all categories of mineral resources may be computed by the geologic blocks method. Accuracy of computations depends, essentially, upon the accuracy of factors accepted for each block and, to a lesser extent, on the accuracy of block area determinations. They may be as accurate as any other method, when a proper number of observations support the computation of factors for a certain deposit. On the other hand, the computations by this method may be speculative or purely academic, when the factors are based on an insufficient number and density of observations.

Examples of computations by this method are quite common in the early stages of exploration of bedded deposits; that is, phosphate rock, limestone, gypsum, and coal (fig. 30). It is often the only method that can be used when the deposit is irregular.

An excellent example of resource computations by geologic blocks has been published by the Geological Survey for uranium and vanadium deposits of the Colorado Plateau (8). The ore bodies are roughly tabular and generally parallel the bedding of the sandstone host rock. They are irregular, often small in size (less than 5,000 tons), of variable thickness with uranium and vanadium values distributed erratically. Computations have been based on drill holes, underground openings, observations of natural outcrops, and production records. Often the number of observations for individual deposits were restricted.

At Lvov-Katin (near Moscow, U.S.S.R.) coal reserve computations were made by both geologic blocks and polygon methods (64). Geologic blocks were selected on the basis of bed thickness, as there were only small variations in coal quality (ash, sulfur, etc.). A total of 15 principal and 80 supplementary geologic blocks were used as compared with 260 blocks by the polygon method. The geologic blocks method revealed the presence of areas of varied and
sharply reduced coal thickness. As the geologic and mining conditions were different from other blocks, this area required additional exploration to permit reserve computations of the same category as the rest of the deposit.

Advantages

The average factors and area methods of reserve computations are relatively simple; their use, however, requires training and experience. Areas are measured by planimeter, computed, or scaled from maps. In general, the factors are determined by a minimum number of simple calculations; no special or detailed maps are needed. The procedure is flexible and requires no complex formulas; computations can be made for individual blocks, panels, levels, segments, or for the entire mineral body.

These methods are adaptable to all types of deposits and to all stages of development; they allow rapid and continuous evaluation of factual data, thus permitting improved engineering planning.

Individual observations of thickness and grade are often unconfirmed with respect to their localities; therefore, computations of average factors usually do not require area weighting. Changes in reserves of a mine, whether due to extraction or continuing exploration, can be easily made by subtracting or adding respective areas, or by determining new or corrected areas.

Accuracy of the computations varies depending on the type of deposit, number of blocks, and density of observations. When a deposit is quite uniform and average factors are computed on the basis of a sufficient number of observations, results are accurate.

Application

Analogous and geologic blocks methods are widely used for all types of mineral deposits. For successful application their principal requirement is a geologic and geochemical similarity between the segment or block being considered, and a more thoroughly studied portion of the same or similar deposit. Both methods are convenient for rapid approximations to support exploration and everyday mining decisions. They often can be used when other methods fail because of lack of sufficient data.

Certain types of mineral deposits, such as tabular, bedded, and large placer deposits, are particularly suited to these methods. Their basic parameters vary only slightly from one point to another, and the average factors may be determined with sufficient accuracy by a simple averaging, regardless of whether observations are distributed in a systematic or unsystematic pattern. Another use of these methods is when the thickness of a mineral body can be accurately measured, but the high cost or technical difficulties make it impossible to sample raw material in place. Grade in such a case may be computed from past production, other portions of the deposit, or even mill and smelter returns.
Both methods should be used with discretion, because the accuracy for a deposit may depend on personal interpretation, rather than objective geologic observations and sampling.

**Mining Blocks Method**

The mining blocks is also known in the mineral industry as longitudinal section, mine extraction, and mine exploitation (11, 28, 31). A mining block may be defined as a portion of a mineral body delineated on four sides by workings, or bounded by workings on three or less sides, and by survey or arbitrary lines on the remaining sides (fig. 31). The size and form of the mining block is determined by exploration and development workings, geologic features, technical, and economic considerations.

In practice, mining blocks are generally

**FIGURE 31.** Mining Blocks Exposed on Four Sides. A, Vertical section of a vein developed by underground workings; B, isometric drawing of three mining blocks a, b, and c (Note-vein thickness less than width of workings); C, geometric interpretation of the same blocks for computations.
rectangular in shape with the bases lying in the plane of a plan, vertical, or incline longitudinal section, depending on the geologic characteristics of the deposit. The most common form of a mining block is parallelepiped; block volume of ore is computed as a product of area and average thickness (formula 15), ore tonnage as a product of volume and weight factor (formula 17), and metal tonnage as a product of ore tonnage and average grade (formula 29). The usual form for reserve computations by this method is given in tables 9 and 10.

Mining blocks delineated by a combination of underground openings and drill holes are special cases. Assignment of reserves into categories depends on the type of deposit, kind and density of mine workings, and economics. Accuracy depends on the way the blocks were delineated and on the method and accuracy of the sampling. Assuming that all workings are studied with the same accuracy, several typical cases are discussed further.

TABLE 9. - Determination of arithmetic average of factors for individual blocks

<table>
<thead>
<tr>
<th>Block</th>
<th>Name of workings</th>
<th>Thickness ((t)_i) (^{\text{ft}})</th>
<th>Grade ((c)_i) (^{\text{percent}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Raise (AB)</td>
<td>(t_1)</td>
<td>(c_1)</td>
</tr>
<tr>
<td></td>
<td>Raise (AB_1)</td>
<td>(t_2)</td>
<td>(c_2)</td>
</tr>
<tr>
<td></td>
<td>Drift (AA_1)</td>
<td>(t_3)</td>
<td>(c_3)</td>
</tr>
<tr>
<td></td>
<td>Drift (BB_1)</td>
<td>(t_4)</td>
<td>(c_4)</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>(\sum t')</td>
<td>(\sum c')</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>(t''_{av} = \sum t''/4)</td>
<td>(c''_{av} = \sum c''/4)</td>
</tr>
<tr>
<td>2</td>
<td>Raise (AB_1)</td>
<td>(t_1)</td>
<td>(c_1)</td>
</tr>
<tr>
<td></td>
<td>Raise (AB_2)</td>
<td>(t_2)</td>
<td>(c_2)</td>
</tr>
<tr>
<td></td>
<td>Drift (AA_2)</td>
<td>(t_3)</td>
<td>(c_3)</td>
</tr>
<tr>
<td></td>
<td>Drift (BB_2)</td>
<td>(t_4)</td>
<td>(c_4)</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>(\sum t'')</td>
<td>(\sum c'')</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>(t'''_{av} = \sum t'''/4)</td>
<td>(c'''_{av} = \sum c'''/4)</td>
</tr>
</tbody>
</table>

\(t\) are determined as average arithmetic thickness by formula (10).
\(c\) are determined as average arithmetic grade by formula (24), or thickness-weighted average grade by formula (25), or area-weighted average grade by formula (26).
TABLE 10. - Recapitulation of reserves for mineral body (by categories) and determination of average grade

<table>
<thead>
<tr>
<th>Block</th>
<th>Area (S), sq ft</th>
<th>Average thickness (t_{av}), ft</th>
<th>Raw material volume (V), cu ft</th>
<th>Weight factor (F), cu ft/ton</th>
<th>Raw material reserves (Q), tons</th>
<th>Valuable components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average grade (c_{av}), percent</td>
</tr>
<tr>
<td>1.....</td>
<td>S_1</td>
<td>t'_{av}</td>
<td>V_1</td>
<td>F</td>
<td>Q_1</td>
<td>c'_{av}</td>
</tr>
<tr>
<td>2.....</td>
<td>S_2</td>
<td>t''_{av}</td>
<td>V_2</td>
<td>F</td>
<td>Q_2</td>
<td>c''_{av}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N.....</td>
<td>S_n</td>
<td>t_{avn}</td>
<td>V_n</td>
<td>F</td>
<td>Q_n</td>
<td>c^n_{av}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total..</th>
<th>( \sum S )</th>
<th>( \sum V )</th>
<th>( \sum Q )</th>
<th>( \sum P )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>i=1</td>
<td>i=1</td>
<td>i=1</td>
<td>i=1</td>
</tr>
<tr>
<td>Average</td>
<td>( \frac{\sum S}{n} )</td>
<td>( \frac{\sum V}{n} )</td>
<td>( \frac{\sum Q}{n} )</td>
<td>( \frac{\sum P}{n} )</td>
</tr>
</tbody>
</table>

Block Exposed on Four Sides by Underground Workings

Reserve computations of a number of mining blocks opened on all sides by underground workings is made by determining average factors for each workings; determining average factors for each block; computing volume, ore, and metal tonnages for each block; and summarizing the reserves of all blocks of the same category and computing weighted average factors for each category and for the entire mineral deposit.

Determining Average Factors for Each Working

When the thickness of a mineral body is uniform and less than the width of underground openings, average factors for each working are usually found by a simple arithmetic average of appropriate variables. Average thickness may be computed by weighting areas of influence of each thickness according to the rule of nearest points, depending on the form and size of the mineral body, irregularities in values, and density and distribution of observations. In irregular bodies average grade for a working is computed by weighting each sample by appropriate areas, volumes, and tonnages.

Determining Average Factors for Each Block

When the thickness of mineral body is less than the width of the opening, average factors are computed as follows:
lengths of all sides are equal,

\[ t_{av} = \frac{t_1 + t_2 + t_3 + t_4}{4} \]  
(33)

\[ c_{av} = \frac{c_1 + c_2 + c_3 + c_4}{4} \]  
(34)

where \( t_1, t_2, t_3, \) and \( t_4 \) are thicknesses measured or computed for each working; \( c_1, c_2, c_3, \) and \( c_4 \) are grades for the same workings.

When lengths of sides are unequal and there is no relationship between thickness and grade, average factors of a mining block may be computed by weighting each working according to its length, \( L_1, L_2, L_3, \) and \( L_4, \)

\[ t_{av} = \frac{t_1 L_1 + t_2 L_2 + t_3 L_3 + t_4 L_4}{L_1 + L_2 + L_3 + L_4} \]  
(35)

and

\[ c_{av} = \frac{c_1 L_1 + c_2 L_2 + c_3 L_3 + c_4 L_4}{L_1 + L_2 + L_3 + L_4} \]  
(36)

When lengths of sides are unequal and thicknesses and grades of workings vary considerably (fig. 31A), average factors are

\[ t_{av} = \frac{t_1 s_1 + t_2 s_2 + t_3 s_3 + t_4 s_4}{s_1 + s_2 + s_3 + s_4} \]  
(37)

and

\[ c_{av} = \text{formula 26}, \]

where \( s_1, s_2, s_3, \) and \( s_4 \) are areas of influences of each working found by the rule of nearest points.

When the ore thickness is more than the width of the mine openings and the blocks are developed by crosscuts on two levels, reserves may be computed from the crosscut data, as discussed further for a block exposed on two sides. Drift and raise samples between crosscuts are not required, if there are evidences of good ore continuity and grade uniformity. In the case of an irregular body the block may be divided into areas of influence to check results of original computations.

**Block Exposed on Three Sides by Underground Workings**

Average factors for a mining block exposed on three sides by underground workings are computed similar to the previous cases: (a) as an arithmetic average of three sides (disregarding length of workings); (b) by weighting variables of each side of the block according to the length of each working; (c) by computing first the factors for the fourth side from end-samples \( a \) and \( b \) of the existing sides (fig. 32A left) and, then, averaging the variables of all sides (this, however, increases the importance of the end-samples in
comparison with others); and (d) by weighting the areas of influence of each of the three existing workings (fig. 32A right).

A block delineated by an adit, raise, and surface workings, that is trenches or pits, is a special case and may be subdivided as above (fig. 32B right), or on the basis of accuracy of computations (category) (fig. 32B left). It may also be subdivided into blocks based on geologic evidences, such as degree of ore alteration, thickness, grade, zoning, or number of observations (32C).

Block Exposed on Two Sides

**Underground Workings on Two Levels**

When a mining block is developed on two levels (fig. 33, block 1) average factors for each level are found first. In uniform bodies the block factors are computed as the average of both levels; otherwise they are computed as area-weighted averages of both levels.

**Intersecting Underground Workings**

A mining block developed on two sides by underground intersecting workings; that is drift and raise, is a triangular prism (fig. 33, block 2). Block factors may be determined as the average of both workings, as the length-weighted average of both
FIGURE 33. - Mining Blocks Exposed on Two Sides (Vein Thickness Less Than the Width of Workings). A, Blocks exposed between two parallel workings (1) and between two intersecting workings (2); B, isometric drawing of the same mining blocks.
workings, or as the area-weighted average of both workings with areas of influence found according to the rule of nearest points (blocks 2a and 2b).

In the first and second examples, volumes are computed as the product of average thickness \( (t_{av}) \) and area \( (S) \); in the third example as a sum of two auxiliary blocks, each calculated as the product of the average thickness of the working and its area of influence.

When the thickness of the mineral body is greater than the width of the workings, the mining block may be developed on both levels by crosscuts or by crosscuts and drifts (fig. 34). In the first example reserves are computed from crosscut data by the vertical cross-section methods. When drift samples between crosscuts are available, the factors and area reserves may be computed by the rule of nearest points for each level. Block reserves are computed by one of the formulas discussed further in the section on cross-section methods.

**Block Exposed on One Level and Intersected at Depth by Drilling**

A mineral body exposed by workings on one level and intersected at depth (or above) by one or more drill holes may be divided into mining blocks by constructing perpendiculars from the drill hole ore intersection level \( a, b, \) and \( c \) to the underground workings level \( a', b', c' \) (fig. 35A). Boundaries of blocks also may be determined on the basis of geologic criteria, such as zoning or rake of mineralization, and on basis of mining design and economics. If thickness and grade of the body are uniform, block factors are computed by

\[
\begin{align*}
t_{av} &= \frac{t_1 L_1 + t_2 L_2}{L_1 + L_2} \\
c_{av} &= \frac{c_1 L_1 + c_2 L_2}{L_1 + L_2}
\end{align*}
\]

where \( t_1 \) and \( c_1 \) are average thickness and average grade of ore in each block on the drift level; \( t_2 \) and \( c_2 \) are average thickness and average grade of each two adjoining drill holes limiting the side of block; and \( L_1 \) and \( L_2 \) are the block lengths on both levels.

When the mineral body is intersected by one drill hole, the mining block may be divided according to the rule of nearest points into two auxiliary blocks of varying accuracy and, therefore, of different categories (fig. 35B).

The average factors may be determined by formulas (57, p. 219),

\[
\begin{align*}
t_{av} &= \frac{3t_1 + t_2}{4} \\
c_{av} &= \frac{3c_1 + c_2}{4}
\end{align*}
\]
FIGURE 34. - Mining Blocks Exposed on Two Sides (Vein Thickness More Than the Width of Workings). A, Plan of two mining blocks delineated by crosscuts; B, block diagram of two mining blocks delineated on two levels by crosscuts.

FIGURE 35. - Mining Blocks Exposed by Drift and Drill Holes. A, Block exposed on one level by a drift and intersected at depth by two or more drill holes; B, block exposed on one level by a drift and intersected at depth by one drill hole.
where \( t_1 \) and \( c_1 \) are average thickness and assay for the drift and 
\( t_2 \) and \( c_2 \) are thickness and assay for the drill hole.

When a large number of samples are available from the drift, drill hole 
information often may serve only to delineate the block and evaluate the min-
eralization. If each observation, whether it is from the drift or drill hole, 
is considered on an equal basis, then the computations of average thickness 
and grade are made by formulas.

In the example of numerous holes the formulas are

\[
\bar{t} = \frac{t_1 + t_2 + \ldots + t_n + t_1' + t_2' + \ldots + t_m'}{n + m} \tag{42}
\]

and

\[
\bar{c} = \frac{c_1 + c_2 + \ldots + c_n + c_1' + c_2' + \ldots + c_m'}{n + m} \tag{43}
\]

With one hole they are

\[
\bar{t} = \frac{t_1 + t_2 + \ldots + t_n + t_1'}{n + 1} \tag{44}
\]

and

\[
\bar{c} = \frac{c_1 + c_2 + \ldots + c_n + c_1'}{n + 1} \tag{45}
\]

In these computations,

\( t_1, t_2, \ldots t_n \) are thicknesses observed in the drift,
\( c_1, c_2, \ldots c_n \) are corresponding assays,
\( t_1', t_2', \ldots t_m' \) thicknesses observed in drill holes,
\( c_1', c_2', \ldots c_m' \) corresponding assays,
n - number of samples in the drift, and
m - number of drill holes.

Application

In order to apply the mining blocks method it is necessary to develop the 
mineral body into blocks (for extraction) by a sufficient distribution of work-
ings. In general, reserves for uniform bodies are of the highest category; 
that is, proved or semipproved. Computations are relatively simple. Block 
reserves may be classified for mining purposes according to thickness, grade, 
and extraction cost. Thus, the method allows the operator to control the 
quality and the cost of production.

The method is flexible and may be used in all types of mineral deposits. 
The degree of error depends, to a great extent, on the genetic type of the 
deposit, on the density of workings, and on the distribution of observations. 
It is naturally adapted to sedimentary beds such as coal, to vertical and 
steeply dipping veins of thin and medium thickness, and to thin-bedded tabular
ore bodies, where grade and thickness undergo gradual changes and where mining and geologic features are similar to blocks already extracted. When weighted averaging of thickness and grade can be eliminated from the computations, it becomes a simple operation.

In nestlike, broken, or interrupted bodies, or where mineral values are distributed erratically, the relative error of this method may be excessive.

Cross-Section Methods

Principles and Requirements

The initial step in the application of cross-section methods is to divide the mineral body into blocks by constructing geologic sections at intervals along the transverse lines or at different levels in conformity with exploration workings, purpose of computations, and the nature of the deposit (figs. 36 and 37). The interval between the sections may be constant or may vary to suit the geology and mining requirements. When the intervals are unequal, formulas for computations are slightly more complicated.

Depending on the manner of the block construction there are three modifications of cross-section methods:

1. **Standard method** based on the rule of gradual changes (fig. 37A). Each internal block is confined by two sections and by an irregular lateral surface, and each end block by a single section and by an uneven lateral surface.

![FIGURE 36. Block Layout by Cross-Section Methods (Block Diagram). A, Rule of gradual changes—standard cross-section method; B, rule of nearest points—linear cross-section method.](image-url)
FIGURE 37. - Cross-Section Methods–Standard and Linear. A, Laying out blocks according to the rule of gradual changes; B, laying out blocks according to the rule of nearest points.

1. Surface. Sections may be parallel or nonparallel, vertical, horizontal, or inclined;

2. **Linear method** based on the rule of nearest points (fig. 37B). Each block is defined by a section and a length equal to one-half the distance to the adjoining sections; and

3. **Method of isolines** based on the rule of gradual changes (see section entitled Method of Isolines).
For accurate results cross-section methods require

(a) that a sufficient number of workings completely crossing the mineral body and an adequate number of observations and samples taken from each section to represent the quality of the raw material;

(b) that the workings lie in or near the sections; and

(c) that all workings are distributed more or less equally between the sections.

Standard Method--Parallel Sections

Procedure

Some earth scientists distinguish two variables of the standard method: vertical or fence used mainly in exploration; and horizontal or level used in mining. The usual procedure for computing reserves by this method is (a) determining the areas of all sections, (b) computing average factors for each section; (c) computing volume, and ore and metal tonnages for each block; and finally, (d) summarizing the results for all blocks by categories and computing average factors for the entire body (table 11). Measurement of individual areas has been discussed in part 1 of this report.

Average factors for each section may be determined by the rule of gradual changes, rule of nearest points (length-weighted average and area-weighted average), and as an arithmetic average. When in the case of irregular distribution of workings, there is a direct or inverted relationship between thickness and grade, average grade may be computed as a thickness-weighted average.

Volume Computations

The configuration of areas and the lateral shape of the blocks are usually irregular and, in order to compute volume by solid geometry, areas are considered to be of equal size circles or polygonal figures; lateral surfaces of the blocks are disregarded.

Mean-Area Formula. - The simplest formula for a volume between two parallel sections with areas $S_1$ and $S_2$ and a perpendicular distance, $L$, between them is

$$V = \frac{(S_1 + S_2)}{2} L.$$ (46)

This mean-area formula is precise when both areas are nearly similar in size and shape.
### TABLE 11. - Computation of reserves by standard method of cross-sections

<table>
<thead>
<tr>
<th>Blocks</th>
<th>Sections</th>
<th>Area (S), sq ft</th>
<th>Interval between sections (L), ft</th>
<th>Volume (V), cu ft</th>
<th>Weight factor (F), cu ft/ton</th>
<th>Raw material reserves (Q), tons</th>
<th>Valuable component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>{ A-A, B-B }</td>
<td>{ S_1, S_2 }</td>
<td>{ L_1 }</td>
<td>( V_1 = \frac{(S_1 + S_2)}{2} \ L_1 )</td>
<td>( F )</td>
<td>( Q_1 = \frac{(S_1 + S_2)}{2} \ L_1 )</td>
<td>( c_{av}^1 = \frac{P_1}{Q_1} )</td>
</tr>
<tr>
<td>2.</td>
<td>{ B-B, C-C }</td>
<td>{ S_2, S_3 }</td>
<td>{ L_2 }</td>
<td>( V_2 = \frac{(S_2 + S_3)}{2} \ L_2 )</td>
<td>( F )</td>
<td>( Q_2 = \frac{(S_2 + S_3)}{2} \ L_2 )</td>
<td>( c_{av}^{11} = \frac{P_2}{Q_2} )</td>
</tr>
<tr>
<td>. . . . . . .</td>
<td>. . . . . . .</td>
<td>. . . . . . .</td>
<td>. . . .</td>
<td>. . .</td>
<td>. . .</td>
<td>. . .</td>
<td>. . .</td>
</tr>
<tr>
<td>N.</td>
<td>{ S_{n-1}, S_n }</td>
<td>{ L_n }</td>
<td>( \frac{(S_{n-1} + S_n)}{2} \ L_n )</td>
<td>( F )</td>
<td>( Q_n = \frac{(S_{n-1} + S_n)}{2} \ L_n )</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

| Total... | \( n \) | \( \sum S \) | \( \sum L \) | \( \sum V \) | \( n \) | \( \sum Q \) | \( n \) | \( \sum P \) |
| Average... | | | | | | | | |
A solid mineral body that has been divided into blocks by a series of evenly spaced parallel sections is computed by the "end-area" formula derived from formula (46),

\[ V = (S_1 + 2S_2 + 2S_3 + \ldots + S_n) \frac{L}{2}, \]  

(47)

where \( L \) equals the distance between sections. If the sections are unevenly spaced the formula for the volume of the entire body will be

\[ V = \frac{(S_1 + S_2)}{2} L_1 + \frac{(S_2 + S_3)}{2} L_2 + \ldots + \frac{(S_{n-1} + S_n)}{2} L_n, \]  

(48)

where \( L_1, L_2, L_3, \ldots L_n \) are perpendicular distances between the adjoining sections with areas \( S_1, S_2, S_3 \ldots S_n \).

**Wedge and Cone (Pyramid) Formulas.** - End blocks of lenslike mineral bodies may be converted to a wedge or cone (pyramid) with the larger areas \( S \) in one section, tapering to a line or a point in the adjoining section (fig. 38A). If the block tapers to a line, volume is computed by wedge formula,

\[ V = \frac{S}{2} L. \]  

(49)

This formula, however, is precise only when the base is rectangular and the lateral faces are isosceles triangles and trapezoids. A more precise formula for the wedge is (42)

\[ V = \frac{L}{6} (2a + a_1) b \sin \alpha, \]  

(50)

where \( a \) and \( b \) are the lengths of sides of the base \( \alpha \) - angle between \( a \) and \( b \), and \( a_1 \) is the larger side of the trapezoid (38A right).

If the block tapers to a point (fig. 38B), volume is computed by cone formula,

\[ V = \frac{S}{3} L. \]  

(51)

Volume computed by the wedge formula is 50 percent larger than volume computed by the cone formula.

**Frustum Formula.** - When \( S_1 \) and \( S_2 \) vary in size, but are similar (fig. 38C), frustum of a cone or pyramid formula is used to compute the block volume,

\[ V = \frac{L}{3} (S_1 + S_2 + \sqrt{S_1 S_2}). \]  

(52)
In practice, the frustum formula is avoided because of complications involved in computing square roots and in certain cases it is less accurate than the prism formula. Let us consider a block of a regular prism (volume of which is $V = S \times L$) and divide it into two auxiliary prisms in the form of truncated wedges with areas $S_1$ and $S_2$ equal to 0.9S and areas $S_3$ and $S_4$ equal to 0.1S (fig. 39) (30).
FIGURE 39. - Frustum Formula Versus Mean-Area Formula.

According to frustum formula the volume of each auxiliary block is

\[ V = \frac{S_1 + S_2}{2}, \quad L = \frac{2.0}{2} L = 1.0 L \]

\[
\begin{align*}
a_1 &= a_2 \\
b_1 &= b_2 \\
S_3 &= S_6 = 0.1 S \\
S_4 &= S_5 = 0.9 S \\
S_1 &= S_2 = S_3 + S_4 = S_5 + S_6 \\
V_1 &= V_2 = \frac{L}{3} \left( 0.1 + 0.9 + \sqrt{0.1 \cdot 0.9} \right) \\
&= \frac{L}{3} \left( 1.0 + 0.9 \right) \\
&= \frac{1.3}{3} L = 0.433 L \\
V &= V_1 + V_2 = 0.866 L
\end{align*}
\]

In this example the total block volume, by frustum formula, is 13.4 percent less than the volume computed by the regular prism formula. It is concluded, therefore, that the frustum formula is inaccurate in wedgelike bodies. Thus, when the areas delineating the truncated wedge blocks have equal sides, such as heights \( b_1 \) and \( b_2 \) between two levels (fig. 40A), or thicknesses \( a_1 \) and \( a_2 \).
between parallel sections (fig. 40B), the mean-area formula is more precise than the frustum.

Prismoidal Formula

Many mineral bodies swell, contract, pinch, and in general, have irregular lateral surfaces that may have a profound influence on the accuracy of volume computations. The prismoidal formula is based on the assumption that the enclosing lateral, curved and warped surfaces can be accurately replaced by plane triangles, trapezoids, or parallelograms bounded by straight lines and constructed from one parallel sector to the adjoining one (10, 14, 20, 40, 42, 62). Selection of the plane figures is controlled by the shape of the mineral body and its enclosing surface.

The prismoidal formula is derived from Simpson's rule for irregular areas.

\[
V = (S_1 + 4M + S_2) \frac{L}{6},
\]

where \( M \) is the area of an auxiliary plane section parallel to and midway between sections \( S_1 \) and \( S_2 \). Construction of this auxiliary section is based on interpolation of longitudinal and cross sections and by interpretation of the geology of the mineral body (39, p. 581, 28, p. 117). Only in exceptional cases is \( M \) an average of \( S_1 \) and \( S_2 \). Auxiliary area constructions require additional work.

This formula is advantageous when a mineral body is divided into blocks by a series of closely spaced cross sections. Alternate or odd number sections may be regarded as end block sections, and each intervening or even
numbered section as a midsection M. The formula is recommended when the cross sections are of different configuration and more accurate computations are desired; it is commonly used in civil engineering for earth work and is described in field surveying handbooks (39, p. 582, 57, v. 1, p. 231).

Volume computations, with the precision of the prismoidal formula, may be made by reducing the results computed by the mean-area formula by a "prismoidal correction factor",

\[ V_{\text{prismoidal}} = V_{\text{mean-area}} - C_{\text{prismoidal correction}} \]

where \( C \), the factor for any triangular prismatoid, is equal to

\[ C = \frac{L}{3} (S_1 - 2M + S_2) = \frac{L}{12} (a_1 - a_2)(b_1 - b_2) \]  

(54)

and is expressed in cubic feet.

\[ M = \frac{(a_1 + a_2)(b_1 + b_2)}{2}, \]

(55)

FIGURE 41.- Blocks Between Parallel Sections. Influence of the shape of areas on volume computations.
where \( a_1 \) and \( b_1 \) are sides of area \( S_1 \) and \( a_2 \) and \( b_2 \) are sides of area \( S_2 \) (fig. 41, Block A). Tables of values for triangular prisms for a distance of 100 feet between sections, and for prismoidal corrections are available in publications related to railway and other earth excavation problems. The obelisk formula, described in some publications as a separate formula (30), is a modification of a prismoidal formula derived by substituting equation (55) for \( M \).

\[
V = \frac{L}{6} \left( S_1 + 4M + S_2 \right) = \frac{L}{6} \left[ S_1 + 4 \left( \frac{a_1 + a_2}{4} \right) \left( b_1 + b_2 \right) + S_2 \right]
\]

\[
= \frac{L}{3} \left[ S_1 + S_2 + \left( \frac{a_1 b_1 + a_2 b_2}{2} \right) \right].
\]

(56)

The influence of the shape of parallel areas on volume computations is well demonstrated by the following. Comparing the results of computations by mean-area, frustum, and prismoidal formulas for two bodies with the same value base areas \( S_1, S_2, S_3, \) and \( S_4 \), but of different shape (fig. 41), gives

<table>
<thead>
<tr>
<th>Distance between sections, feet</th>
<th>15</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions of bodies, feet:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_1 )</td>
<td>16</td>
<td>-</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>80</td>
<td>-</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>80</td>
<td>-</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>-</td>
<td>16</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>-</td>
<td>80</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>-</td>
<td>40</td>
</tr>
<tr>
<td>( b_4 )</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>-</td>
<td>80</td>
</tr>
<tr>
<td>Mean-area, ( \text{ft}^3 )</td>
<td>1,200</td>
<td>1,200</td>
</tr>
<tr>
<td>Frustum, ( \text{ft}^3 )</td>
<td>1,200</td>
<td>1,200</td>
</tr>
<tr>
<td>Prismoidal, ( \text{ft}^3 )</td>
<td>1,380</td>
<td>2,425</td>
</tr>
<tr>
<td>Relative errors of mean-area and frustum formulas, percent</td>
<td>13.0</td>
<td>50.5</td>
</tr>
</tbody>
</table>

Thus, in our example the volumes of blocks A and B computed by the mean-area and frustum formulas are 13.0 and 50.5 percent less than the values computed by prismoidal formula.

If values \( b_1 \) and \( b_2 \) are equal, the prismoidal (obelisk) formula converts to the mean-area formula.

\[
V = \frac{L}{3} \left[ S_1 + S_2 + \left( \frac{a_1 b_2 + a_2 b_2}{2} \right) \right] = \frac{L}{3} \left[ S_1 + S_2 + \left( \frac{S_1 + S_2}{2} \right) \right] = \frac{L}{2} \left( S_1 + S_2 \right)
\]
Thus, when any pair of side values of \( a \) and \( b \) are equal or nearly equal, the mean-area formula is accurate. This is in general a common case of volume computations, such as a block having equal height or thickness.

The prismoidal formula has been used in computations of complex vein ore bodies in the Coeur d'Alene mining district, Idaho (10).

**Bauman's Formula.** - Among the less common formulas for volume computations is one offered by Bauman (2, 3). This graphic formula has limited use.

When the lateral and irregular surface of a mineral body between two parallel sections is assumed to be linear (fig. 42), the volume of the block may be computed by

\[
V = \frac{L}{2} (S_1 + S_2) - \frac{LR}{6} \quad \text{or} \quad V = \frac{L}{6} (3S_1 + 3S_2 - R),
\]  

(57)
where $S_1$ and $S_2$ are areas of cross sections, 
$L$ is distance between sections, and 
$R$ is an auxiliary area, graphically constructed as follows:

1. Draw both end areas and construct projections ($AA_{11}^{11}$, $BB_{11}^{11}$, ...) from the generators ($AA^1$, $BB^1$, ...) of the lateral surface on the same plane (figs. 42B and 42C).
2. From point $O$ (fig. 42D) construct lines of equal length and direction as the lines $AA_{11}^{11}$, $B_{11}^{11}$, ... If the generators are taken in sufficient quantity and all points $A$, $B$, $C$, ... are connected by a curved line, the resultant figure will be the auxiliary area $R$.

Construction of auxiliary areas requires experience and time, therefore, limiting the use of the formula. The common prism and pyramid formulas may be derived from Bauman's formula by making appropriate assumptions. When area $S_1$ is equal to $S_2$, the auxiliary area $R$ is zero and Bauman's formula is the same as a prism or cylinder. When area $S_2$ is zero, the auxiliary area $R$ will be equal to $S_1$, and Bauman's formula is that of a pyramid.

**Tonnage Computations**

The product of block volume and weight factors produces the tonnage of raw material; the product of the latter and average grade equals the reserves of valuable component. Another manner of tonnage computation for each block consists of determining "section reserves" for a slice of one unit in width, and computing block reserves as the product of half the sum of the section reserves and the block length.

Section reserves are computed as the total of the reserves between workings and may be determined by the rule of gradual changes (fig. 43A) or the rule of nearest points (fig. 43C). Section reserves are often termed linear reserves when taken along exploration lines. The term "linear reserves", in this report, is set apart for reserves computed for one square unit of area; that is, square foot, square yard, etc. (fig. 43B), and the term "area reserves" is used for areas of substantial size such as acre and square mile.

**The Standard Method for Nonparallel Sections**

Sections constructed along exploration lines may converge or diverge because of changes in the strike of the mineral body. The angles between sections and exploration lines may range from oblique to obtuse, depending on strike variations. Formulas offered for computing reserves with nonparallel sections are discussed in the following sections.

**Angle Less Than 10 Degrees**

When the angle of intersection is less than 10 degrees, Zolotarev offered the formula (fig. 44) (27, 31, 43, 57, 66),

$$v = \frac{(S_1 + S_2)}{2} \left( \frac{b_1 + b_2}{2} \right),$$

(58)
FIGURE 43. - Section and Linear Reserves—Cross-Section Methods. A, Layout of auxiliary blocks according to the rule of gradual changes; B, linear reserves per square unit; C, layout of auxiliary blocks according to the rule of nearest points.
Center of gravity for area $S_2$

Center of gravity for area $S_1$

$A$

Center of gravity for area $S_2$

Center of gravity for area $S_1$

$B$

FIGURE 44. - Standard Cross-Section Method for Volume Computations—Nonparallel Sections.

$A$, Graphic representation of mineral body crossed by nonparallel sections; $B$, plan-construction of perpendiculars $h_1$ and $h_2$ from center of gravity of one section to the other.

where $S_1$ and $S_2$ are areas of the mineral body in the sections, and $h_1$ and $h_2$ are the lengths of two respective perpendiculars dropped from the center of gravity of one section to another.

\[
V = \begin{cases}
\frac{(S_1 + S_2)(h_1 + h_2)}{2} & \text{Angle} \ < 10^\circ \\
\frac{a}{\sin \alpha} \left( \frac{(S_1 + S_2)}{2} \right) \left( \frac{h_1 + h_2}{2} \right) & \text{Angle} \ > 10^\circ
\end{cases}
\]
In practice reserves computed by the previous formula deviate slightly from those computed by mean-area formula; in most cases the use of the Zolotarev formula for a block between sections with an angle of intersection less than 10 degrees is not necessary.

**Angle Greater Than 10 Degrees**

When the angle of divergence between the sections is greater than 10 degrees, a correction factor \( \frac{\alpha}{\sin \alpha} \), is used by Zolotarev. The angle between sections is expressed in radians.

\[
V = \frac{\alpha}{\sin \alpha} \frac{(S_1 + S_2)(h_1 + h_2)}{2}.
\]

This formula and the preceding one are considered accurate when the sizes of the sectional areas do not differ more than four to six times.

Some scientists consider the Zolotarev formulas accurate only in a case of a fragment of a ring-shaped body with the center coinciding with the intersection of sections. In their opinion the use of the above formulas leads to systematic error, more often increasing rather than decreasing the results (48).

The formulas require location of the centers of gravity of each area, which is difficult in complex geometric figures. The best procedure of finding the center of gravity of an area is graphical, when the coordinates of the center are determined by the sum of the static moments. Such determinations are inconvenient and cumbersome and a more practical procedure consists of reproducing the area in cardboard and locating its center of gravity by hanging with a thread.

Compared with parallel sections computations of volumes and tonnages by nonparallel formulas may show appreciable differences; variations in average grade, however, are relatively small. A simpler method of computing reserves for blocks bounded by nonparallel sections is discussed in the next chapter.

**Linear Method**

In the linear cross-section method of reserve computations, blocks are constructed according to the rule of nearest points (figs. 36B and 37B); each block rests on one section with the length of influence extending half the distance to the adjoining sections. Ore, \( Q \), and metal reserves, \( P \), are usually determined as the product of linear ore and metal reserves \( q_L \), \( p_L \), and area \( A \),

\[
Q = q_L A \quad (60)
\]

and

\[
P = p_L A. \quad (61)
\]
When ore and metal reserves \( q_v \) and \( p_v \) are given per cubic unit, volume instead of area is required in previous formulas,

\[
Q = q_v V
\]

and

\[
P = p_v V.\]

For an illustration of the use of the linear method let us take a block between two nonparallel sections (fig. 45). The ore block, two, for the area of influence \( A_2 \) of section 2-2, lying between nonparallel sections 1-1 and 3-3, is found by bisecting the angles \( \alpha \) and \( \beta \) with auxiliary sections \( e-e \) and \( e_1-e_1 \). Reserve computations are made by formulas,

\[
Q_2 = q_2 (A_{2}^1 + A_{2}^{11})
\]

and

\[
P_2 = p_2 (A_{2}^1 + A_{2}^{11}),
\]

where \( A_{2}^1 + A_{2}^{11} = A_2 \) (area of influence of section 2-2);

\( q_2 \) - are linear ore reserves (per square foot), determined for section 2-2 by dividing section reserves on the length of the body along the section;

\( p_2 \) - are linear metal reserves determined similar to \( q_2 \).

The linear method is suitable for computing reserves of placer deposits, where exploration is carried out in stages; exploration lines are drawn across the changing course of the deposit, and workings are distributed equally along such a line. If additional lines of exploration are added between the initial ones, the distances between sections and the areas of influence will decrease and construction of the appropriate blocks will change. Reserves will remain unchanged, unless new workings are added.

Advantages

The cross-section methods graphically portray the geology of the mineral deposit. The general procedure is simple and rapid, but formulas producing greater precision may necessitate use of diagrams, additional calculations, and construction of auxiliary sections. To increase the accuracy of computations the number of blocks should be as large as possible; in other words, the sections should be placed close together.

Care should be exercised to avoid arbitrary locations and construction of sections. Distance between sections is usually governed, in exploration, by the character of the mineral body and the distribution of mineral values. Selection of sections unjustified by exploration data may influence the size of the areas and, in turn, the computations. Construction should not rely on interpretation made over distances unmerited for the given type of deposit. Most of the disadvantages in the use of this method can be avoided by properly planned exploration.

Computations of two or more ore bodies in the sections are possible.
Application

Cross-section methods are being used effectively when construction of sections is possible with a minimum amount of interpolation and extrapolation. The application of various formulas depends on the analysis of the layout of the sections, on the relative size and shape of ore areas, and on the distances between sections.

Common formulas for volume computations are mean-area and frustum. The regular prism formula requires equality in size and shape of both areas. The mean-area formula is accurate when the areas in parallel sections are similar; it should not be used when side dimensions a and b are different. If the areas of parallel sections differ by more than 40 percent, the frustum formula should be used.

The prismoidal formula is accurate for all various forms of solids. Bauman's formula requires construction of an auxiliary area. The proper use of the various formulas for different geometric solids is given in table 12. In general, computation of block volumes by the methods of cross sections requires analysis of the shape and size of sections to determine the best formula, particularly in conjugated blocks.
Well-defined and large bodies that are uniform in thickness and grade or have gradually changing values can generally be computed accurately by cross-section methods. Sections may be vertical, inclined, or horizontal, as in pipelike or stock deposits. Two suites of sections constructed at right angle to each other may be employed for large mineral bodies with more or less evenly distributed values, such as stocks and impregnations. The final results may then be computed as an average of both suites of sections, or one suite may serve for control of the other.

The method should be used with discretion in all cases where the bodies are irregular, or where values tend to concentrate in ore shoots. When computations of several valuable components are required and the mineral body shows grade variations for each component, it is difficult and often impossible to apply cross-section methods. Horizontal cross sections, constructed along levels or horizons of workings, is preferred because of the selection or design of mining method.

Quite often it is necessary to compute reserves of ore between levels, or of different grade within a block, separately. The sum of such auxiliary computations should be equal to the block reserves; absence of such control indicates a possible source of error.

Cross-section methods are easily adaptable for use simultaneously with other methods. Reserves, developed in upper levels by underground workings, may be computed by the mining blocks method, and reserves of lower levels, explored by drilling, by the standard cross-section method. Numerous examples of such computations are described in the literature (60-61, 65).

### TABLE 12. - Application of various formulas in computing volumes of solid bodies in standard cross-section method

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>Solid bodies between parallel sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean-area......</td>
<td>$L \frac{(S_1 + S_2)}{2}$</td>
<td>X</td>
</tr>
<tr>
<td>Frustum........</td>
<td>$\frac{1}{3} (S_1 + S_2 + \sqrt{S_1 \times S_2})$</td>
<td>X</td>
</tr>
<tr>
<td>Prismoidal (Simpson)</td>
<td>$\frac{1}{6} (S_1 + 4S_2)$</td>
<td>X</td>
</tr>
<tr>
<td>Obelisk.........</td>
<td>$\frac{1}{3} \left[ S_1 S_2 + \frac{(a_1 b_2 + a_2 b_1)}{2} \right]^3$</td>
<td>X</td>
</tr>
</tbody>
</table>

Reported as a new formula by Kravchenko and Kupfer.

Source: Reference (30).
Method of Isolines

Principles and Formulas

Isolines are curved lines joining all points of equal unit value. They are used to graphically illustrate natural physical and chemical properties or processes that can be expressed by unit values. A common example is a topographic map, where relief is expressed by contours of equal elevation. Isolines are widely accepted in earth and engineering sciences for visual delineation and distribution studies of various physical and chemical phenomena. Well-known applications are maps using lines to depict similar relationships such as isothermal, isostatic, isomagmatic, isopach (or isothickness), isocal (isocalorific values for coal), isocarb (equal content of fixed carbon), isograde, and others. Less common are complex isolines such as linear reserves (foot-percent, tons, per square foot, or dollars per square foot) used in computing reserves of mineral deposits.

The method is based on the assumption that unit values, from one point to another, undergo continuous and uninterrupted changes according to the rule of gradual changes. To construct isolines, intermediate values are determined by interpolation between points of known values; as a result certain properties of mineral bodies may be presented graphically on a plan or section by a system of isolines. In aggregate such a system constitutes an imaginary surface similar to a topographic map. These "toposurfaces" are graphic expressions of numbers and, thus, may be used according to the principles of solid and analytical geometry.

The theory of the method of isolines for use in mining and engineering was developed by Sobolevsky (37, 53). He disclosed that the toposurfaces can be added, subtracted, multiplied, and divided and that even more complex operations, such as extraction of roots, involution, differentiation, and integration, could also be made. Practical applications of this method in geology and mining are varied. Detailed discussion of these applications is beyond the scope of this paper.

Common cases are computations of average thickness, average grade, and average value of a mineral deposit from appropriate isoline maps (6, 11). Only an isopach map is needed to compute volume and tonnage of mineral ore reserves. A unit of volume reserves at a given point on such a map is a product of height, equal to the thickness of the body, and area, equal to a unit value (square foot, square yard, square meter, etc.). A unit of tonnage ore reserves is a division of volume reserves and volume-tonnage factor. To compute the weight of metal or other valuable component in the deposit, isolines of linear metal reserves (product of linear ore reserves and grade) are constructed.

Let us examine a portion of an isopach map (fig. 46). The mineral body, confined in nature by irregular surfaces, is transformed for computations to an equivalent body limited on one side by a flat plane base and on the other by a complex surface represented by a series of isolines of equal thickness or height. Thus, the isopach map gives a distorted picture of the mineral
FIGURE 46. - Method of Isolines. A, Cross section of a mineral body along exploration line 10-10; B, isopach plan of the same mineral body along the exploration line 10-10; C, cross section along exploration line 10-10 made from isopach plan B.
body. Each isoline on the map may be considered a projection of a slice of the mineral body, divided by a series of parallel equally spaced sections. The volume and tonnage of each slice may be computed by standard cross-section formulas; namely, mean-area (46) and frustum (52). Areas bounded by appropriate isolines are measured by planimeter. The distances between the areas are constant, equal to the isopach interval. The volume of the entire mineral body is a sum of all slices (47).

When the mineral body is irregular in thickness, the computations may be complicated. If the area of thickness $h_2$ is in two portions, $S_2^1$ and $S_2^{11}$, the volume of the slice between $h_1$ and $h_2$ will be (fig. 47A)

$$V_2 = h \frac{S_1 + (S_2^1 + S_2^{11})}{2}.$$  

(64)

When thickness $h_2$ is missing from part of the area $S_2$, but has instead a thickness $h_1$, the volume of a slice between the $h_2$ and $h_1$ isolines will be

$$V_3 = \frac{h}{2} [(S_2^1 + S_2^{11}) + (S_3^1 - S_3^{11})],$$

(65)

where $h$ - constant thickness interval between isolines;

$S_0$ - area enclosed by $h_0$ contour line (minimum thickness);

$S_1$ - area enclosed by $h_1$ contour lines; and

$S_2^1$, $S_2^{11}$, and $S_3^{11}$ - areas enclosed by $h_2$ contour line.

Average ore grade may be computed in much the same way by the construction of isograde maps (fig. 47B), and by weighting areas outlined by isolines for each grade; namely

$$c_{av} = \frac{c_0 A_0 + \frac{c}{2} (A_0 + 2A_1 + 2A_2 + \ldots + A_n)}{A_0},$$

(66)

where $c_0$ is the minimum grade of ore;

$c$ - constant grade interval between isolines;

$A_0$ - area of ore body with grade $c_0$ and higher;

$A_1$ - area of ore body with grade $c_0$ plus $c$ and higher;

$A_2$ - area of ore body with grade $c_0$ plus $2c$ and higher, etc.

The isograde map graphically illustrates the grade distribution of the ore.

The metal tonnage reserves (or other valuable constituents) may be found by multiplying toposurfaces of isopach and isograde maps and constructing linear metal reserves maps (weight factor constant). The geometric meaning of the linear metal reserves may be well illustrated by an imaginary ingot, received by settling at right angles all metal particles on the plane of the map. The metal reserves of the mineral body then will be confined between base plane and linear metal toposurface. By slicing the ingot by sections parallel to the base on equal distances, traces of such intersections with toposurface will yield the isolines of linear metal reserves.
Requirements, Advantages, and Limitations

The method of isolines, also known as level plan, is a graphical modification of the horizontal standard cross-section method. It is distinctive and, therefore, is discussed in a separate chapter. The method requires a sufficient number, appropriate density, and distribution of observations for accurate plotting of isolines. When observations are unevenly distributed, the weight influence of one station may not be the same as for the others; in some deposits the density of observations may be insufficient; and in others it may exceed the required accuracy. The method is appropriate for mineral bodies where there are certain natural regularities in the variations in thickness, grade, and value.

A major advantage of the method is its descriptive- ness. The isopach map gives an idealized likeness of the mineral body, second only to the model. The isograde map shows the distribution of rich and poor ore, and the map of linear reserves illustrates the distribution of reserves of raw material and valuable constituents. Isoline maps are

\[
\text{General formula} \quad c_{av} = \frac{c_0 A_0 + \frac{c}{2} (A_0 + 2A_1 + 2A_2 + \ldots + A_n)}{A_0}
\]

\[
\text{Formula for figure B} \quad c_{av} = \frac{c_0 A_0 + \frac{c}{2} [A_0 + 2A_1 + 2(A_2' + A_2'') + (A_3' + A_3'')]}{A_0}
\]

FIGURE 47. - Isopach and Isograde Maps for Reserve Computations—Method of Isolines. A, Isopach map; B, isograde map.
easy to read, measure, and interpolate; calculations are replaced by graphic interpretations and there are fewer blocks, instead of numerous small blocks used in some methods.

The method permits better mine planning. The boundaries of cutoff ore are easily constructed and changed. Volume can be computed by measuring areas of respective isolines without additional drawing. If the requirements for minimum grade, thickness, or value of ore are changed, the isomaps remain the same; reserve computations can be made plus or minus one or several slices of the mineral body. The method can be automated through calculating machines or computers and, when properly applied, prevents discrepancies between the original reserve estimates and those remaining after partial extraction.

There are several disadvantages in the use of the method. First, the position of the isolines depends on the scale of the map, interval accepted, density of workings, and accuracy of construction. Second, in practice, dissimilar toposurfaces can be constructed on the basis of the same data by different persons. Construction and interpolation of data have an influence on the size of areas and, therefore, on the final results of computations. Two or more solutions are possible. When data are profuse, construction and interpretation of isolines may be complex or even cumbersome. Such is the case in multimetal deposits. Checking computations is intricate and may take as much time as a complete reestimate of reserves. In general, the method of isolines is best applied to deposits where thickness and grade decreases from center to periphery.

Application

The method of isolines is widely used for illustration and analysis of physical and chemical properties of mineral deposits. Isoline maps are often irreplaceable in studies of the morphology of mineral bodies; geochemical and geophysical prospecting are also aided by the application of this method.

The use of isolines for reserve computations is limited mainly to deposits showing orderly changing thickness and grade characteristics. The method is widely used in computing volumes for earthwork operations (39, p. 583), stockpiles (44), and computing reservoir reserves of water, natural gas, and oil (3, pp. 91-98). It is time consuming and nonoperative when the grid of exploration workings is sparse. It is impractical in structurally broken and small ore bodies, in complex multimetal and very irregular mineral deposits.

Isolines are known to be drawn from interpretation of aerial photographs for computing volume of material removed from the area, as well as ore reserve in stockpiles. Few examples of the use of this method for reserves computations are described in literature (26, 56).
Method of Triangles

Principles and Formulas

A plan (or longitudinal section) of the mineral deposit showing exploration workings and the entire area of the mineral body can be divided graphically into a system of triangles by connecting holes with straight lines (figs. 48A and B). Each triangle on the plan represents a horizontal projection or a base area of an imaginary prism with edges--$t_1$, $t_2$, $t_3$--equal to vertical thicknesses of the mineral body; the upper base is truncated. Thus, the mineral body is divided into a series of close order triangular prisms with base areas in a plane of the map (figs. 48C and D). Hence, the name of the method of triangles or triangular prisms. The rule of gradual changes of all variables from one working to another is the main principle of this method. In figure 48A, the layout of the triangles involves the use of data from hole 1 eight times; data from holes 2 through 9 are used twice each. The layout in figure 48B involves the use of data from holes 3 and 6 five times, hole 9 four times, hole 1 three times, holes 5 and 8 two times, and holes 2, 4, and 7 once each.

The formula for computing volume of a truncated triangular prism with uneven heights of edges is

$$ V = \frac{1}{3} (t_1 + t_2 + t_3) S. \quad (67) $$

The average grade of each prism is usually determined as an arithmetic average of $c_1$, $c_2$, and $c_3$, or as the thickness-weighted average of the same grades; that is,

$$ c_{av} = \frac{c_1 + c_2 + c_3}{3} \quad (68) $$

or

$$ c_{av} = \frac{c_1 t_1 + c_2 t_2 + c_3 t_3}{t_1 + t_2 + t_3}. \quad (69) $$

Ore tonnage for a triangular block is computed by

$$ Q = \frac{V}{F} \text{ or } \frac{(q_1 + q_2 + q_3)}{3} S. \quad (70) $$

Metal tonnage for a triangular block is computed by

$$ P = Q c_{av} \text{ or } \frac{(q_1 c_1 + q_2 c_2 + q_3 c_3)}{3} S \text{ or } \frac{(p_1 + p_2 + p_3)}{3} S, \quad (71) $$

where $q_1$, $q_2$, and $q_3$ are linear ore reserves of the edges of the triangular prism (in tons); $c_1$, $c_2$, and $c_3$ are the grades of the edges of each triangular prism (in percent); and $p_1$, $p_2$, and $p_3$ are linear metal reserves of the edges of the triangular prism (in tons).
Volume, ore and metal tonnages of the entire mineral body, and average grade are computed by the usual formulas: volume - (16), ore tonnage - (18), metal tonnage - (29a), gravimetric average grade - (28), and volumetric average grade - (27).

Some earth scientists prefer to compute average grade by the gravimetric formula for individual blocks, and by the volumetric formula for the entire mineral body. The latter is convenient when a weight factor is accepted as constant for the entire body.

Procedure

Laying out the Triangles

All workings on the maps are connected with straight lines and the area of the mineral body is divided into a maximum number of triangles; no line should be crossed by another. For accurate computations the ideal triangle is equilateral; a circle circumscribed through the vertices of the triangle should be the smallest possible. The common practice is to take the shortest diagonal of each trapezium area. Theoretical investigations support such a choice (see next section).

Some scientists prefer to select triangles corresponding to the morphology of the mineral body. In vein deposits, for example, the bases of the triangles may be extended along the strike, so that all triangles have a uniform slope. For placers, Baxter and Park suggest finding the configuration of the mineral body with isopachs and then constructing triangles with a uniform slope throughout each triangle (2, p. 46).

The number of triangles and lines are constant for each project. A simple way to check the correctness of layout is by formulas (27),

\[ N_t = 3n_1 + 2n_2 + 3, \text{ or } N_L = 3(n_1 - 1) + 2n_2 \]  
(72)

for the number of connecting lines and

\[ N_{tr} = 2n_1 + n_2 - 2 = 2(n_1 - 1) + n_2 \]  
(73)

for the number of triangles,

where \( n_1 \) is the number of workings inside the perimeter of the mineral body, and

\( n_2 \) - the number of workings bounding the body.

Determining Areas of Individual Triangles

If the area of any triangle is computed as one-half the product of the base line and height, it is desirable to use the common line of two contiguous triangles as a base for computing the other area.

The procedure of computing volume, ore and metal tonnages, and average grade is illustrated in tables 13 and 14.
### TABLE 13. - Determination of average arithmetic grade for each triangle by triangular method

<table>
<thead>
<tr>
<th>Workings numbers</th>
<th>Triangles numbers</th>
<th>Area (S), sq ft</th>
<th>Thickness (t), ft</th>
<th>Average thickness (tavr), ft</th>
<th>Volume (V), cu ft</th>
<th>Weight factor (F), cu ft/ton</th>
<th>Raw material reserves (Q), ton</th>
<th>Grade (c), percent</th>
<th>Valuable component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3, 4</td>
<td>1 S₁</td>
<td>t₁, t₂, t₃</td>
<td>t₁ + t₂ + t₃ / 3</td>
<td>V₁</td>
<td>F</td>
<td>Q₁</td>
<td>(c₁, c₂, c₃)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1, 2, 3, 4</td>
<td>11 S₁₁</td>
<td>t₁, t₂, t₃</td>
<td>t₁ + t₂ + t₃ / 3</td>
<td>V₁₁</td>
<td>F</td>
<td>Qₑ₁</td>
<td>(c₁, c₂, c₃)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>c₁ + c₂ + c₃ / 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Average**

Σ S / i=1 | Σ V / Σ S / i=1 | Σ Q / i=1 | Σ P / i=1

### TABLE 14. - Determination of thickness-weighted average grade for each triangle by triangular method

<table>
<thead>
<tr>
<th>Workings numbers</th>
<th>Triangles numbers</th>
<th>Area (S), sq ft</th>
<th>Thickness (t), ft</th>
<th>Average thickness (tavr), ft</th>
<th>Volume (V), cu ft</th>
<th>Weight factor (F), cu ft/ton</th>
<th>Raw material reserves (Q), ton</th>
<th>Grade (c), percent</th>
<th>Product</th>
<th>Valuable component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3, 4</td>
<td>1 S₁</td>
<td>t₁, t₂, t₃</td>
<td>t₁ + t₂ + t₃ / 3</td>
<td>V₁</td>
<td>F</td>
<td>Q₁</td>
<td>(c₁, c₂, c₃)</td>
<td></td>
<td>c₁ t₁ + c₂ t₂ + c₃ t₃ / (t₁ + t₂ + t₃)</td>
<td>P₁ = c₁avr Q₁</td>
</tr>
<tr>
<td>1, 2, 3, 4</td>
<td>11 S₁₁</td>
<td>t₁, t₂, t₃</td>
<td>t₁ + t₂ + t₃ / 3</td>
<td>V₁₁</td>
<td>F</td>
<td>Q₁₁</td>
<td>(c₁, c₂, c₃)</td>
<td></td>
<td>c₁ t₁ + c₂ t₂ + c₃ t₃ / (t₁ + t₂ + t₃)</td>
<td>P₁₁ = c₁₁avr Q₁₁</td>
</tr>
</tbody>
</table>

**Average**

Σ S / i=1 | Σ V / Σ S / i=1 | Σ Q / i=1 | Σ P / i=1

n / n / n / i=1
Studies by Different Authors

For many years the triangular method was considered standard, although errors in results due to the manner of dividing the area into triangles were recognized. The unreliability of this method was discussed by Harding in 1921 (13-14) and by Zhuravsky in 1934 (68-69). The latter studied the relative error for a volume of a block, explored by four vertical holes, with thicknesses $t_1$, $t_2$, $t_3$, and $t_4$ and base area $S$ (fig. 49). The volume of the right prism may be computed in two ways: $V_1$—as the volumes of two triangular prisms with bases ABD and BDC; or $V_2$—as volumes of two triangular prisms with bases ABC and ADC; that is

$$V_1 = \frac{1}{3} (t_1 + t_2 + t_s) \frac{S}{2} + \frac{1}{3} (t_2 + t_3 + t_4) \frac{S}{2}$$

$$= \frac{1}{6} (t_1 + 2t_2 + t_3 + 2t_4) S$$

$$V_2 = \frac{1}{6} (2t_1 + t_2 + 2t_3 + t_4) S.$$  

In the first case $t_2$ and $t_4$, and in the second case $t_1$ and $t_3$, are taken twice in each formula. Graphically, the lower surface of the block in the first case is convex and the volume is overestimated, and in the second, the block is concave and the volume is underestimated.

$\text{Lower surface convex}$  

$\text{Lower surface concave}$

FIGURE 49. - Two Manners of Construction of Triangular Prisms for a Rectangular Prism.
The volume of the prism is computed by half the sum of both cases,
\[ V = \frac{V_1 + V_2}{2}, \] or
\[ \frac{1}{2} \left[ \frac{1}{6} \left( t_1 + 2t_2 + t_3 + 2t_4 \right) S + \frac{1}{6} \left( 2t_1 + t_2 + 2t_3 + t_4 \right) S \right] = \frac{1}{4} \left( t_1 + t_2 + t_3 + t_4 \right) S. \]

The latter is a standard rectangular prism formula.

The relative error between volumes \( V_1 \) and \( V_2 \) is
\[ \Delta V = V_1 - V_2 = \frac{1}{6} \left( t_1 + 2t_2 + t_3 + 2t_4 \right) S - \frac{1}{6} \left( 2t_1 + t_2 + 2t_3 + t_4 \right) S \]
or
\[ \Delta V = \pm \frac{S}{6} (t_1 - t_2 + t_3 - t_4). \] (74)

If \( \Delta V \) is equal to zero, the volumes \( V_1 \) and \( V_2 \) are equal, and
\[ t_1 + t_3 = t_2 + t_4. \] (75)

In other words, the method of triangles is accurate only if the sum of the two opposite edges, \( t_1 \) and \( t_3 \), of each rectangular prism are equal to the two remaining edges, \( t_2 \) and \( t_4 \). Assuming that \((t_1 + t_3)\) is two times less than \((t_2 + t_4)\); that is \(2 (t_1 + t_3) = (t_2 + t_4)\), the volume of \( V_1 \) will be more than volume \( V_2 \) for the value of \( \frac{S}{12} (t_2 + t_4) \), a relative error of 20 percent.

Modifications

In the triangular method equal weights are given to each hole in the triangular prism; this is correct only in a prism with an equilateral base. In several modifications of this method the average grades of each triangular prisms are determined by weighting ore thicknesses (formula 25), angles of the triangle (13-14), side lengths of each triangle (57), distances of each hole from the center of gravity (15), and areas of influence on each hole, constructed by rule of nearest points.

The "Harding angular system" of computations was an attempt to reduce the relative error inherent in the method by applying a factor equal to \( \frac{180^\circ}{1/3} \) in the volume formula for each hole of the triangle instead of a constant factor of 1/3. In other words, he subdivided the truncated triangular prism into three auxiliary prisms with heights equal to the edges of the basic prism and areas weighted according to the magnitude of the angle of the basic prism.

Later, Harding found the system to be incorrect in computing average grade, particularly when the prism is apexed on a hole having a zero value thickness. The computed grade may be of higher tenor than any of the other values from the holes of the prism. To correct such errors the following formulas are given by Harding (13, p. 124):
When all holes have positive values,

\[ t_{av} = \frac{t_1 + \frac{t_1 + t_2}{2} + \frac{t_1 + t_3}{2}}{3} = \frac{4t_1 + t_2 + t_3}{6}. \]  

(76)

Similar formulas are used for holes 2 and 3.

When \( t_3 \) has zero value,

\[ t_{av} = \frac{3t_1 + t_2}{6}; \]  

(77)

if \( t_2 \) and \( t_3 \) have zero values,

\[ t_{av} = \frac{t_1}{3}. \]  

(78)

Additional studies of the triangular method led Harding and others to the development of the polygonal method.

When the workings are distributed in a regular grid and the areas of the triangles are equal or nearly equal, reserves may be computed by these simplified formulas \((27, 31)\),

\[ V = \frac{1}{3} s \sum_{i=1}^{N} t_k, \]  

(79)

\[ Q = \frac{1}{3} s \sum_{i=1}^{N} t f_k, \]  

(80)

and

\[ P = \frac{1}{3} s \sum_{i=1}^{N} t f c k, \]  

(81)

where \( s = \frac{S}{N} \)

- \( S \) - total area of all triangles
- \( N \) - number of triangles
- \( t \) and \( f \) - thicknesses and weight factors of triangles
- \( k \) - coefficient determined by the number of triangles starting at each hole.

This variation may be used to facilitate computations in cases of large numbers of triangles; errors connected with the construction of triangles and their measurements are eliminated and the results do not depend on the manner of constructing the triangles.
Distinctive Features

The method of triangles is basically formal and withdrawn from geologic and mining considerations. The inside perimeter of the net of triangular prisms may be in conflict with the physical boundaries of the body, and the prism sides may cross the boundaries of individual ore types. It is often difficult or even impossible to subdivide the ore body into segments of similar thickness or grade. Triangles may conceal the distribution of variables.

The procedure for reserve computations by the method is relatively simple, when the formula for truncated prisms is used. Modifications of the method require more elaborate computations. The relative error depends on the manner in which the area is divided into triangles, their form, and the total number of triangles.

In comparison with other methods the triangle method requires construction of a greater number of blocks ultimately resulting in labor and time consuming computations. When a mineral body contains several valuable components, computations may also be cumbersome.

In the triangular method of computations the use of exploration data concerning individual mine workings may not be constant, for example, in figure 48A data for holes on the perimeter of the body are used two times in comparison with eight times within the body (hole 1). In cases of irregular sharp changes in variables, inside workings may have a disproportionate influence on the computations.

The method is not exact when variables decrease from the center to the outside boundaries, such as the thickness of lenslike bodies. According to Zhuravsky the volume reserves of a lenticular body computed by this method will be underestimated (68).

Thus, errors in computing reserves may be substantial, particularly when fluctuations of variables are large and the number of triangles is small. When mine workings are numerous and closely spaced, errors for each triangle tend to compensate each other. Even in the most favorable distribution of workings, such as square set, the triangular method may produce an appreciable error.

Application

The uniform and gradual changes of variables are characteristic features of only a few mineral deposits, predominantly sedimentary. Naturally, the triangular method, based on the rule of gradual changes of interpretation of exploration data, is most applicable to such deposits. Large sedimentary and large disseminated ore deposits, explored by regularly spaced drill holes, have been computed by this method (58).
Method of Polygons

Principles

The method of polygons, also known as polygonal prisms, equal spheres of influence, areas of equal influence, and areas of nearest points, is based on the concept that all factors, determined for a certain point of a mineral body, extend half the distance to adjoining and surrounding points, thus forming an area of influence. The rule of nearest points was discussed in part 1 of this report. Briefly, areas of equal influence are found for workings symbolized on the map as points by using perpendicular bisectors and for those symbolized as lines by angle bisectors.

The first description of the method was given as early as 1909 by Boldyrev (57). In the United States the method was developed independently from the triangular method by Davis (13, p. 122) and Harding (14) during the 1920's. A concept of areas of equal influence was introduced step by step, and it was accepted and developed as a new principle for the polygon method, where triangles are used as auxiliary constructions. The first application of this method in the United States was in computing reserves of extremely irregular bodies of the Joplin and Wisconsin zinc deposits in 1920 (13).

Procedure and Construction of Polygons

In this method, the explored portion of the mineral body is substituted by a series of polygonal prisms, the plane bases being equal to areas of influence of appropriate workings (fig. 50). Each such prism assumes the thickness, weight factor, and grade determined for such workings.

The usual steps in computing reserves by this method are

1. Construction of auxiliary triangles, when necessary; the manner of construction of triangles has no influence on the final shape of polygons;

2. Construction of polygons by following a definite order; for example, clockwise and from periphery to the center of the deposit;

3. Computing reserves for each block (table 15);

4. Grouping of blocks on the basis of evaluation of grade, thickness, linear reserves, reliability, etc., and summarizing and classifying reserves into various categories.

**TABLE 15. - Computation of reserves by polygonal method**

| Polygon number | Area (S), sq ft | Thickness (t), ft | Volume (V), cu ft | Weight factor (F), cu ft/ton | Raw material reserves (Q), tons | Valuable component 
|----------------|----------------|------------------|-------------------|-----------------------------|-------------------------------|-------------------------
| 1............. | S₁             | t₁               | V₁                | F                           | Q₁                            | c₁  P₁                   
| 2............. | S₂             | t₂               | V₂                | F                           | Q₂                            | c₂  P₂                   
| 3............. | S₃             | t₃               | V₃                | F                           | Q₃                            | c₃  P₃                   
| ...           | ...            | ...              | ...               | ...                         | ...                           | ...                     
| N............. | Sₙ             | tₙ               | Vₙ                | F                           | Qₙ                            | cₙ  Pₙ                   
| Total...      | Σ S i=1        | Σ t i=1          | Σ V i=1           | Σ F i=1                     | Σ Q i=1                       | Σ c i=1 Σ P i=1          
| Average       | \( t_{av} = \frac{\Sigma V}{\Sigma S} \) | \( \frac{\Sigma V}{\Sigma S} \) | \( \frac{\Sigma V}{\Sigma S} \) | \( \frac{\Sigma V}{\Sigma S} \) | \( \frac{\Sigma V}{\Sigma S} \) | \( \frac{\Sigma V}{\Sigma S} \) |

**Case of Vertical Holes**

The polygons around vertical holes are constructed by the intersection of perpendicular bisectors erected from the middle of the sides of the triangles. The criteria for correctness is that all points, in a certain polygon, are nearer to the rallying hole than to others. In polygon construction some perpendiculars may not be used at all (triangle ABD, fig. 14) and others may assist construction by their continuation outside the boundaries of the obtuse triangle (triangle BCD, fig. 14).
When the intersection of perpendiculars for two adjoining triangles forms a tetragon, the diagonal line connecting each pair of perpendiculars becomes a side of a polygon. Point $O$ is equidistant from $A, D,$ and $C$ and point $O'$ is equidistant from $A, B,$ and $C.$ The diagonal is equidistant from $A$ and $C.$ The distinctive feature of a correctly constructed polygon is that each of the inside angles between the sides are always less than $180^\circ$.

If the angle bisector manner of construction is used in the above example, the property of nearest points is not fulfilled, because most of the auxiliary triangles are not equilateral (fig. 14). In comparison with the perpendicular bisector manner of construction, angle bisectors produce polygons with twice the number of sides. Polygons may be irregular in shape and have internal angles of more than $180^\circ$; therefore, they are more difficult to measure with a planimeter. Further analysis shows that polygons constructed by angle bisectors are only another graphic expression of the method of triangles.

In short, construction of polygons by perpendicular bisectors for vertical workings satisfies the principle of nearest points, is simpler, and always the same; area measurements are more accurate.

It is possible to construct an area of influence for a given point by a circle when workings are too widely spaced to safely assume continuity of the mineral body. Circle radius is chosen as optimum for a certain category of reserves and a given type of deposit. In such cases the block is in the form of a cylinder instead of a polygonal prism.

**Case of Linear Workings**

When a mineral body is explored by workings represented as lines on a plan, that is, drifts, horizontal drill holes, and trenches, the angle bisector manner of construction of areas of influence is used. A rectangular block is divided into four areas of influence, or four elementary prisms, each one characterized by appropriate workings. The block volume is the sum of all elementary prisms, and the thickness and average grade are computed by weighting the volumes and tonnages of the auxiliary prisms. This modification of the polygon method was previously described as a common case of the mining blocks method, when areas of influence are determined for four sides of a block (fig. 31A, block d).

**Principal Formulas**

**Irregular Distribution of Drill Holes**

Let us first consider a common case where a mineral body is explored by irregularly spaced vertical drill holes. The general formulas for a group of polygonal prisms (fig. 50) are for volume - formula 16, for average thickness - similar to formula 37, for ore tonnage - formula 18, for metal tonnage -

$$P = c_1 q_1 + c_2 q_2 + c_3 q_3 + \ldots + c_n q_n,$$

(82) for average grade - formula 29.
Regularly Spaced Drill Holes

More simple formulas may be derived from principal ones when workings are laid down in a regular grid to form simple polygons, such as squares, rectangles, or hexagons. The common nature of these modifications is that the areas of influence for each hole, except those lying on the boundary of the body, are equal in size.

Square Net of Workings. - The volume of an area, s, explored by four holes with thicknesses \( t_1 \), \( t_2 \), \( t_3 \), and \( t_4 \) is

\[
V = \frac{(t_1 + t_2 + t_3 + t_4)}{4} s. \tag{83}
\]

The area of mineral bodies that have been explored by numerous holes located in the corners of squares will be divided by perpendicular bisectors into squares with equal areas, s; the formula for volume computation will transform to

\[
V = \frac{t_1 + t_2 + t_3 + \ldots + t_n}{N} s = \frac{(t_1 + t_2 + t_3 + \ldots + t_n)}{S} s, \tag{84}
\]

where \( t_1 \), \( t_2 \), \( t_3 \), \ldots, \( t_n \) are thicknesses of holes, and

\[
s = \frac{S}{N}.
\]

When the quantity of holes is limited or more accurate results are desirable, the above formula may be improved by adding to each variable a factor based on true areas of influence in arbitrary units, termed "weight." Thus, in a square network, the area of influence of a hole for a complete square must have a weight of four, for a side hole a weight of two, and for a corner hole a weight of one (fig. 51A).

Chessboard Grid. - In a chessboard or triangular grid map, the entire area of the mineral body will be divided by perpendicular bisectors into hexagons with equal areas (fig. 51B). Formulas for computing volume will be the same as a square grid, except--s will equal the area of a hexagon. When the number of workings is limited, the formula should have a weight of six for a complete, three for a half, and one and a half for a corner of a hexagon.

Requirements, Advantages, and Limitations

The method of polygons is based on theoretical assumptions rather than on geologic and mining considerations and requires a suitable plan or longitudinal section. The correct manner of constructing areas of influence requires experience; however, there is only one way to do it, and the results do not depend on personal judgment. In comparison with other methods the nature of the mineral deposit is poorly illustrated, although, polygons may, under appropriate pattern of exploration workings for a given type of deposit, indicate reasonably well the distribution of thick and narrow and high- and low-grade portions of the body.
The factors, thickness, grade, and weight, are considered constant for each block. Hence, each block is computed without influence from any adjoining blocks, and it is possible to add and compute reserves for new blocks as exploration progresses. In other methods new data often calls for a complete recalculation of reserves.

When the workings are in a regular grid pattern, reserve computations are simplified. The size of the polygons varies when workings are unevenly spaced. More widely spaced holes may have an undue influence on the size of blocks and on the average grade. Any one block may have an unreasonably large influence on the final computations, if the variables of this block vary strongly from the variables of the others.

In the case of irregular distribution of workings it is necessary to measure each polygon with a planimeter. When the blocks are numerous such measurements may be time consuming.

**Application**

Favorable criteria for the use of the method of polygons are the proven continuity of a mineral body between workings and the gradual changes of all variables. The method is best applied when the workings are numerous and in grid pattern. The greater the number of blocks and the more regular the grid, the more accurate are the computations. Reserves of tabular bodies (beds, blankets, and thick veins), large lenses, and stocks are successfully computed by this method (43, 45).
Polygons can be used with discretion in cases of nonuniform and irregularly shaped mineral bodies; they are incorrect when the bodies cannot be correlated satisfactorily between workings, when they are small and distributed erratically (small stocks, ore shoots, chimneys, and pipes), or when horses of waste are present. In the case of small lenslike bodies the middle blocks may show an unduly large influence on the final results. In mineral deposits composed of several bodies overlying each other separate sets of polygons may be constructed for each one.

**Combined Methods**

The use of two or more methods to compute reserves for the same deposit is a common practice. Various methods may be applied for different parts of a body depending on the geology, mine design, type and density of exploration workings, and category of reserve computations. Mining blocks, for example, may be used for high category reserves and geologic blocks for lower categories. A second method may often be used for control of the computations made by the principal method, so that no crude errors may occur.

A common case of combined methods is when one method is applied to outline and divide the mineral body into blocks and another to determine the parameters of each block.

Methods of mining blocks, geologic blocks, and cross sections have been used for high-grade nonferrous veins developed by underground workings, and the polygon method for disseminated ore bodies (copper-zinc deposits in Butte, and copper-nickel deposits in Noril, U.S.S.R.). The Corrigan-McKinney Steel Company, Michigan and Minnesota, computed the reserves of No. 1 ore body by the average factors and area methods and the No. 2 ore body by the cross-section methods (28, p. 147).

At the Inspiration mine, Arizona, disseminated copper ore reserves based on drilling were computed by polygons (quadrilateral prisms) and checked by the cross-section methods (60). At the Ray Copper mine, Arizona, large reserves of irregular bodies of disseminated ore, were computed by the mining blocks method for the portion of the ore body developed for underground mining and by vertical cross sections for the portion explored by churn and diamond drilling (61).

At the Kennecott Copper mine, Bingham Canyon, Utah, the method of horizontal cross sections was used to separate each proposed bench of the open pit development of a large low-grade disseminated copper body. Mining blocks were used for computing reserves developed by underground workings and polygon method for areas explored by vertical churn drill holes. The triangular prism method was used for computing reserves explored by churn drills below underground workings (58).

At the Copper Queen mine, Bisbee, Ariz., "ore in sight" reserves of an irregular and lenticular limestone-replacement body were computed by horizontal cross sections and probable reserves by two sets of vertical cross sections constructed at right angles to each other. A low-grade copper-porphyry
deposit, explored by churn drill holes, was computed by the vertical cross-section method and checked by the triangular method (50).

A combination of horizontal cross-section and polygon methods was used for the design of open pit operations for several large disseminated copper deposits (Berkeley pit, Butte, and Konrad, U.S.S.R.). Horizontal cross sections were drawn for each proposed bench, and reserves within each of the two levels were computed by the polygon method. A combination of horizontal cross sections and geologic blocks for each level also has been used for multimetal deposits (Kadain, U.S.S.R.).

Owing to the strict application of the rule of nearest points to the construction of polygons, the morphology and other peculiarities of mineral bodies, such as patterns or relationships between the elements, may be overlooked. A mineral body may show stability in thickness, uniformity of grade, or definite relationships between grade and thickness along strike better than down the dip. Gold and other heavy minerals in placer deposits may show to a greater degree gradual changes in distribution and grade in one direction than in another. Ash content in coal deposits may increase or decrease in a certain direction owing to paleographic conditions.

The grade of the ore in replacement deposits often depends on structural and lithological characteristics of the country rock. In such cases the method of polygons may be modified; holes may be first connected with auxiliary lines on the basis of geomorphology and other peculiarities, such as strike, dip, or rake of the ore body. Square and rectangular blocks may then be formed by construction of lines parallel to and/or perpendicular to auxiliary lines. This modification is often described in literature as a method of rectangles, or various areas of influence in contrast with the polygon or equal areas of influence method.

Examples of reserve computations published as rectangular methods are classified in this report according to the accepted principles of interpretation of exploration data; as mining blocks method, in underground mining; and as a simple modification of the polygon method, in a regularly spaced grid of drill holes.

In irregularly spaced drill hole grids, the rectangular block construction is found to be subjective, or affected by personal opinion. The use of such construction shows the same disadvantages and the same limitations as the method of triangles.

Thus, the rectangular system may be considered as a combined method, if block construction is made on the basis of geologic, mining, and economic considerations, rather than on plain geometrical points of view, and the factors are computed by arithmetic average, by thickness-weighted average, or by area-weighted average, determined according to the rule of nearest points.
Source of Errors in Reserve Computations

Combined average error in any reserve computations may be expressed by

\[ M_{av} = \sqrt{m_{r}^2 + m_{e}^2 + m_{i}^2}, \]

where \( M_{av} \) - average relative error of reserve computations;
\( m_{r} \) - average relative error of technical errors;
\( m_{e} \) - average relative error of the method and formula used;
\( m_{i} \) - average relative error due to the interpretation of exploration data.

Random errors due to precision of observations compensate each other; they are small in comparison with errors of interpretation and may be disregarded. Biased errors, due to inexperienced personnel, equipment defects, and improper techniques of observations and analyses, may be appreciable and should be compensated for by correction factors prior to computations (part 1).

The comparative accuracy of various methods of computing reserves has been discussed in many publications (2, 46, 49, 53). According to foreign sources the variance in reserves of the same high category computed by different methods for a deposit rarely exceed 10 percent (appendix C).

When the results of computations by a selected method are within 1 to 5 percent of results received by other methods, they are considered by Stammberger to be accurate (59).

In general, the average relative error of the method and formula used for reserve computations should lie within the same limits and should not exceed the average relative error for determining grade, as well as other factors; otherwise such errors will distort the precision of the results of exploration.

In most cases these relative errors due to method are neglected, because the errors of interpretation are much larger and determine the accuracy of computations. The latter errors depend on the type and form of the deposit, on degree of variation in thickness, grade, and other factors, on the kind of exploration workings, their density, and on sample technique.

SUMMARY

Reserve computations of a mineral deposit are a geologic and engineering problem; it is often an intricate task. Selection of a method depends on the geology of the mineral deposit, the kind and density of workings, the appraisal of geologic and exploration data, and the accuracy required. Time and cost of computations are often important considerations.

Knowledge of the mineral deposit's geology is a prerequisite to any reliable computation. This knowledge includes space location, size, shape, environment, country rock, overburden, and hydrology; average grade and distribution of valuable and detrimental constituents; and mineral, chemical and physical characteristics of the raw material.
Accurate computations of a certain deposit require a properly selected and executed exploration program; that is kind of workings, drilling or underground system, number and density of observations, sampling procedure (location, sample interval, and weight of samples), and accurate measurements, analyses, and tests.

To select the best method careful analysis of geology and exploration should be made. In general, the method (or combination of methods) selected should suit the purpose of computations and the required accuracy; it should also best reflect the character of the mineral deposit and the performed exploration. In a complex or irregular deposit, it is advisable to use two or more methods for better accuracy and self-confidence. An average of these methods may be accepted as a final result, or the values of one method may be considered as a control of others.

The purpose of reserve computations is one of the most important considerations in selecting a method. For preliminary exploration the method should best illustrate the deposit, the operations, and permit sequential computations and appraisal. On the other hand, time-consuming procedures should be avoided if reserves are being computed for prospective planning.

The system of mining, or the problem of selecting one, may influence the preference. A certain method of computation may facilitate more so than others the design of development and extraction operations owing to technical and economic factors (mining by levels, average grade, different cutoff, etc.). This explains why, in practice, methods of mining blocks and cross sections are preferred.

The principles of interpretation of exploration data and the analytical perfection of formulas are also considered in method selection. The principles that essentially uphold the described conventional methods are:

<table>
<thead>
<tr>
<th>Method</th>
<th>Principles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analogous</td>
<td>Geologic inference.</td>
</tr>
<tr>
<td>Geological blocks</td>
<td>Do.</td>
</tr>
<tr>
<td>Mining blocks</td>
<td>Mining and other considerations.</td>
</tr>
<tr>
<td>Standard cross section and isolines</td>
<td>Rule of gradual changes.</td>
</tr>
<tr>
<td>Triangular prisms</td>
<td>Do.</td>
</tr>
<tr>
<td>Linear cross section</td>
<td>Rule of nearest points.</td>
</tr>
<tr>
<td>Polygonal prisms</td>
<td>Do.</td>
</tr>
</tbody>
</table>

All formulas for computing volumes, tonnage, and average factors are approximate, because of the irregular size and shape of the mineral body, errors in substituting natural bodies by more simple geometric ones, geologic interpretation, assumptions, and inconsistency in the variables. Accuracy of the final results usually depends more on geologic interpretation and assumptions rather than on the method used. Systematic exploration and uniform
sampling generally simplify the selection and the use of the conventional methods and produce greater accuracy. Reserves of the same category computed by different methods and based on the same data, usually differ slightly.

The average factors and area methods are widely used by earth scientists. In the analogous method, reserves of a block of a deposit can be computed with reasonable accuracy by analysis of results of exploration, development, and extraction (past production) from adjoining blocks of the same or even geologically similar deposits. In this method an individual block of the same or geologically similar deposit may be computed on the basis of limited, or even a single observation properly taken. In the geologic block method, the mineral body is subdivided into segments and blocks essentially on the basis of geology; average factors for each segment or block are determined according to available data by various methods.

The mining blocks method requires adequate data to allow subdivision of the mineral body into blocks either proved or semiproved for extraction. Most often it is used in mining thin and medium-thick veins and tabular bodies.

The cross-section methods are the most convenient ways for computing reserves of uniform mineral deposits. In the standard cross-section method the mean-area formula of a prism with base areas in parallel sections is the most common one; it is precise when there is no large difference in size or shape of base areas. In case of disparity between base areas of more than percent, frustum or prismatic formulas are used.

In underground mining, horizontal cross sections constructed along the proposed mining levels are often preferred in mine design. Two sets of vertical sections at right angles to each other would better illustrate stocklike bodies than any other method. Computations may be made with the final results taken as half the sum of both suites.

The linear cross-section method, where reserves are first determined for a unit of area, unit of volume, or for the sections, is used with advantage in bedded and placer deposits.

The method of isolines requires numerous observations with data more or less regularly distributed in the vertical or horizontal plane of the mineral deposits. It is applicable to deposits of gradual physical and chemical changes, such as sedimentary deposits. Large placer gold deposits, explored by hundreds of shallow pits or drill holes, may be well illustrated and evaluated by the method of isolines.

The analytical methods (triangular and polygonal prisms) are deficient in exposing the morphology of the mineral body and the fluctuations of variables within the individual blocks. Although average thickness and grade are computed, the pattern of their space distribution is not revealed.

The method of triangles is applicable to a few predominantly sediment deposits, the mineralization of which is consistent with the rule of gradual
changes. The method must be carefully applied. Errors of computations may be very high owing to irregularities in variables and unsystematic exploration.

The polygonal method is successfully used in computing reserves of tabular deposits, such as sedimentary beds of coal, phosphate rock, and oil shales; blanket-type, large lenses; and thick vein bodies. The accuracy of the results increases with the number of blocks and the density of the grid of workings and drill holes.


Titles enclosed in parenthesis are translations from the language in which the item was published.

Transliteration of the Russian is according to the system given in the GPO Style Manual.


34. McLaughlin, D. H. Geologic Factors in the Valuation of Mines. Econ. Geol., v. 34, No. 6, 1939, pp. 589-621.


APPENDIX A. - ENGLISH AND METRIC SYSTEMS AND CONVERSION FACTORS

| English (standard) system | Metric system
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Conversion factor to metric system</td>
</tr>
<tr>
<td>Linear Measure</td>
<td></td>
</tr>
<tr>
<td>Foot</td>
<td>0.3048 m</td>
</tr>
<tr>
<td>Yard (3 ft)</td>
<td>0.9144 m</td>
</tr>
<tr>
<td>Mile (5,280 ft)</td>
<td>1.6093 km</td>
</tr>
</tbody>
</table>

| Land or Area Measures   |                               |                           |
| Square foot             | 0.0929 sq m                | Square meter              | 10.764 sq ft               |
| Square yard (9 sq ft)   | 0.8361 sq m               | Square kilometer         | 0.3861 sq mi               |
| Acre (43,560 sq ft)     | 0.4047 hectare            | Hectare                  | 2.4710 acres               |
| Square mile (640 acres) | 2.5900 sq km              |                           |

| Volume Measure          |                               |                           |
| Cubic feet              | 0.0283 cu m                 | Cubic meter              | 35.314 cu ft               |
| Cubic yard              | 0.7646 cu m                |                           | 1.3079 cu yd               |

| Weight Measure          |                               |                           |
| Ounce$^1$               | 28.350 g                     | Gram                    | 0.0353 oz                  |
| Pound                   | 0.4536 kg                    | Kilogram                | 2.2046 lb                  |
| Ounce (troy)            | 31.103 g                     | Metric ton              | 1.1023 short ton           |
| Pound (troy)            | 0.3732 kg                    |                           | 0.9842 long ton            |
| Ton, short (2,000 lb)   | 0.9072 metric ton           |                           |                           |
| Ton, long (2,240 lb)    | 1.0160 metric ton           |                           |                           |

$^1$ Avoirdupois weights when not noted.

APPENDIX B. - USAGE OF VARIOUS METHODS FOR RESERVE COMPUTATIONS FOR SOLID MINERAL DEPOSITS IN U.S.S.R. (percentage of totals)

TABLE B-1. - Computation by type of solid mineral deposits

<table>
<thead>
<tr>
<th>Method</th>
<th>Ore deposits</th>
<th>Nonmetallic deposits</th>
<th>Coal and oil shale deposits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geological blocks</td>
<td>37</td>
<td>46</td>
<td>69</td>
</tr>
<tr>
<td>Mining blocks</td>
<td>12</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cross sections</td>
<td>48</td>
<td>37</td>
<td>1</td>
</tr>
<tr>
<td>Isolines</td>
<td>-</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Polygons</td>
<td>2</td>
<td>14</td>
<td>30</td>
</tr>
<tr>
<td>Triangles</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

1As considered by the All Union Committee of Mineral Reserves, U.S.S.R., for 1941-61.

Source: Reference 57, table 22, p. 205.

TABLE B-2. - Solid mineral deposits by selected years

<table>
<thead>
<tr>
<th>Method</th>
<th>1941-47</th>
<th>1951</th>
<th>1954</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geological blocks</td>
<td>16</td>
<td>12</td>
<td>36</td>
</tr>
<tr>
<td>Mining blocks</td>
<td>34</td>
<td>24</td>
<td>15</td>
</tr>
<tr>
<td>Cross sections</td>
<td>14</td>
<td>45</td>
<td>46</td>
</tr>
<tr>
<td>Polygons</td>
<td>22.5</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>Triangles</td>
<td>2.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Rectangular</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Others</td>
<td>11.0</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

APPENDIX C. - A COMPARISON OF RESERVE COMPUTATIONS MADE BY VARIOUS METHODS (U.S.S.R.)

TABLE C-1. - Polymetal deposit in Altai, U.S.S.R.\(^1\)

<table>
<thead>
<tr>
<th>Methods of computations</th>
<th>Ore</th>
<th>Copper</th>
<th>Lead</th>
<th>Zinc</th>
<th>Gold</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness-weighted average</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Average factors and area (arithmetic average)</td>
<td>100</td>
<td>85</td>
<td>105</td>
<td>102</td>
<td>91</td>
<td>90</td>
</tr>
<tr>
<td>Cross sections</td>
<td>91</td>
<td>97</td>
<td>77</td>
<td>87</td>
<td>90</td>
<td>99</td>
</tr>
<tr>
<td>Polygon</td>
<td>99</td>
<td>104</td>
<td>86</td>
<td>94</td>
<td>99</td>
<td>106</td>
</tr>
<tr>
<td>Triangles</td>
<td>95</td>
<td>104</td>
<td>91</td>
<td>94</td>
<td>94</td>
<td>90</td>
</tr>
<tr>
<td>Triangles modified by the areas of influence for each working</td>
<td>95</td>
<td>99</td>
<td>79</td>
<td>87</td>
<td>94</td>
<td>94</td>
</tr>
</tbody>
</table>

\(^1\)Data for 26 holes--3 holes crossed high-grade lead ore of narrow width.

Source: Reference 46, table 3, p. 29.

TABLE C-2. - Bauxite deposit in Tichvin district, U.S.S.R.\(^1\)

<table>
<thead>
<tr>
<th>Average factors and area (arithmetic average)</th>
<th>Index</th>
<th>Cross sections</th>
<th>Polygons</th>
<th>Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross sections</td>
<td>103.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polygons</td>
<td>99.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangles</td>
<td>97.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^1\)Data for 3 bauxite deposits explored by 41 holes.

Source: Reference 46, table 4, p. 29.
APPENDIX D. - FORMULAS

Main Elements

1. Relationship between true, horizontal, and vertical thicknesses.
   \[ t_{tr} = t_{h} \sin \beta = t_{v} \cos \beta \]

2. True thickness - correction, when \( \alpha = 0 \).
   \[ t_{tr} = t_{hp} \sin (\beta + \theta) \]

3. Horizontal thickness - correction, when \( \alpha = 0 \).
   \[ t_{h} = t_{hp} \frac{\sin (\beta + \theta)}{\sin \beta} \]

4. Vertical thickness - correction, when \( \alpha = 0 \).
   \[ t_{v} = t_{hp} \frac{\sin (\beta + \theta)}{\cos \beta} \]

5. General case - true thickness
   \[ t_{tr} = t_{hp} (\cos \alpha \sin \beta \cos \theta + \sin \beta \sin \theta) \]

6. General case - horizontal thickness
   \[ t_{h} = t_{hp} (\cos \alpha \cos \theta - \cotan \beta \sin \theta) \]

7. General case - vertical thickness
   \[ t_{v} = t_{hp} \cos \theta (\cos \alpha \tan \beta + \tan \theta) \]

Rule of Gradual Changes

8. \[ AC = \frac{(t_{c} - t_{1})}{(t_{2} - t_{1})} AB \]

9. \[ tc = \frac{AC}{AB} (t_{2} - t_{1}) + t_{1} \]

Computations

10. Average thickness
    \[ t_{av} = \frac{t_{1} + t_{2} + t_{3} + \ldots + t_{n}}{n} \]
11. Area - Trapezoid formula

\[ S = \left(\frac{a + b}{2}\right) h \]

12. Trapezoidal rule

\[ S = h \left[ \frac{(a_1 + a_n)}{2} + a_2 + a_3 + \ldots + a_{n-1} \right] \]

13. Simpson's rule

\[ S = \frac{1}{3} h \left(a_1 + 2 \sum a_{\text{odd}} + 4 \sum a_{\text{even}} + a_n\right) \]

14. Volume for a block

\[ V = LBT \]

15. Volume for the entire body

\[ V = V_1 + V_2 + V_3 + \ldots + V_n = t_1 S_1 + t_2 S_2 + t_3 S_3 + \ldots + t_n S_n \]

16. Weight - ore tonnage

\[ Q = \frac{V}{F} \text{ and } Q = Vf \]

18. \[ Q = Q_1 + Q_2 + Q_3 + \ldots + Q_n = V_1 f_1 + V_2 f_2 + V_3 f_3 + \ldots + V_n f_n \]

19. \[ Q = VD \]

20. \[ D_{\text{sat}} = \frac{D_2 (1 - F_o)}{(1 - M_o)} \]

21. \[ F_{s.t.} = \frac{2,000}{62.5} \text{ ft}^3/\text{s.t. or } F_{l.t.} = \frac{2,240}{62.5} \text{ ft}^3/\text{L.t.} \]

22. \[ f = \frac{2,000}{F_{s.t.}} \text{ or } \frac{2,240}{F_{l.t.}} \text{ or } 62.5 \text{ D} \]

23. \[ Q = V_1 D_1 + V_2 D_2 + V_3 D_3 + \ldots + V_n D_n \]

Grade

24. Arithmetic average

\[ c_{\text{av}} = \frac{c_1 + c_2 + c_3 + \ldots + c_n}{n} \]
25. Thickness - Weighted average

\[ c_{av} = \frac{c_1 t_1 + c_2 t_2 + c_3 t_3 + \ldots + c_n t_n}{t_1 + t_2 + t_3 + \ldots + t_n} \]

26. Area - Weighted average

\[ c_{av} = \frac{c_1 S_1 + c_2 S_2 + c_3 S_3 + \ldots + c_n S_n}{S_1 + S_2 + S_3 + \ldots + S_n} \]

27. Volumetric average

\[ c_{av} = \frac{c_1 V_1 + c_2 V_2 + c_3 V_3 + \ldots + c_n V_n}{V_1 + V_2 + V_3 + \ldots + V_n} \]

28. Gravimetric average

\[ c_{av} = \frac{c_1 Q_1 + c_2 Q_2 + c_3 Q_3 + \ldots + c_n Q_n}{Q_1 + Q_2 + Q_3 + \ldots + Q_n} \]

29a. Weight-metal tonnage

\[ P = Q \cdot c_{av} \]

29b.

\[ c_{av} = \frac{P}{Q} \]

30. Correction factor for grade

\[ E = \frac{C_y}{C_x} \]

Errors

31. Errors for the entire body

\[ M_{av} = \frac{M_1 + M_2 + M_3 + \ldots + M_n}{N} \]

32.

\[ M_{av} = \frac{M_1 P_1 + M_2 P_2 + \ldots + M_n P_n}{NP} \]

Mining Blocks Method

33.

\[ t_{av} = \frac{t_1 + t_2 + t_3 + t_4}{4} \]

34.

\[ c_{av} = \frac{c_1 + c_2 + c_3 + c_4}{4} \]
35. \[ t_{av} = \frac{t_1L_1 + t_2L_2 + t_3L_3 + t_4L_4}{L_1 + L_2 + L_3 + L_4} \]

36. \[ c_{av} = \frac{c_1L_1 + c_2L_2 + c_3L_3 + c_4L_4}{L_1 + L_2 + L_3 + L_4} \]

37. \[ t_{av} = \frac{t_1s_1 + t_2s_2 + t_3s_3 + t_4s_4}{s_1 + s_2 + s_3 + s_4} \]

38. \[ t_{av} = \frac{t_1L_1 + t_2L_2}{L_1 + L_2} \]

39. \[ c_{av} = \frac{c_1L_1 + c_2L_2}{L_1 + L_2} \]

40. \[ t_{av} = \frac{3t_1 + t_2}{4} \]

41. \[ c_{av} = \frac{3c_1 + c_2}{4} \]

42. \[ t_{av} = \frac{t_1 + t_2 + \ldots + t_n + t_{11} + t_{12} + \ldots + t_{1m}}{n + m} \]

43. \[ c_{av} = \frac{c_1 + c_2 + \ldots + c_n + c_{11} + c_{12} + \ldots + c_{1m}}{n + m} \]

44. \[ t_{av} = \frac{t_1 + t_2 + \ldots + t_n + t_{11}}{n + 1} \]

45. \[ c_{av} = \frac{c_1 + c_2 + \ldots + c_n + c_{11}}{n + 1} \]

Cross Sections Method

46. Mean-area formula
\[ V = \frac{(S_1 + S_2)}{2} L \]

47. End-area formula (equal distances between sections)
\[ V = (S_1 + 2S_2 + 2S_3 + \ldots + S_n) \frac{L}{2} \]

48. Volume for entire body (unequal distances between sections)
\[ V = \frac{(S_1 + S_2)}{2} L_1 + \frac{(S_2 + S_3)}{2} L_2 + \ldots + \frac{(S_{n-1} + S_n)}{2} L_n \]
49. Wedge formula

\[ V = \frac{S}{2} L \]

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50. More accurate wedge formula

\[ V = \frac{L}{6} (2a + a_1) b \sin \alpha \]

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51. Cone formula

\[ V = \frac{S}{3} L \]

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52. Frustum formula

\[ V = \frac{L}{3} (S_1 + S_2 + \sqrt{S_1 S_2}) \]

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53. Prismoidal formula

\[ V = (S_1 + 4M + S_2) \frac{L}{6} \]

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54. Prismoidal correction factor - C (for triangular prism)

\[ C = \frac{L}{3} (S_1 - 2M + S_2) = \frac{L}{12} (a_1 - a_2) (b_1 - b_2) \]

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55. Value of M for prismoidal formula

\[ M = \frac{(a_1 + a_2)}{2} \left( \frac{b_1 + b_2}{2} \right) \]

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56. Obelisk formula

\[ V = \frac{L}{6} (S_1 + 4M + S_2) = \frac{L}{6} \left[ S_1 + 4 \left( \frac{(a_1 + a_2)}{2} \left( \frac{b_1 + b_2}{2} \right) \right) + S_2 \right] \]

\[ = \frac{L}{3} \left[ S_1 + S_2 + \frac{(a_1 b_2 + a_2 b_1)}{2} \right] \]

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57. Bauman's formula

\[ V = \frac{L}{2} (S_1 + S_2) - \frac{LR}{6} \text{ or } V = \frac{L}{6} (3S_1 + 3S_2 - R) \]

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The Standard Method for Nonparallel Sections

58. Angle less than 10°

\[ V = \frac{(S_1 + S_2)}{2} \left( \frac{h_1 + h_2}{2} \right) \]

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59. Angle greater than 10°

\[ V = \frac{\alpha}{\sin \alpha} \left( \frac{(S_1 + S_2)}{2} \left( \frac{h_1 + h_2}{2} \right) \right) \]

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Linear Method

60. Ore tonnage (based on linear ore reserves)
\[ Q = q_l A \]

61. Metal tonnage (based on linear metal reserves)
\[ P = p_l A \]

62. Ore tonnage (based on ore reserves per cubic unit)
\[ Q = q_v V \]

63. Metal tonnage (based on metal reserves per cubic unit)
\[ P = p_v V \]

Method of Isolines

64. Volume
\[ V_2 = h \frac{S_1 + (S_2^1 + S_2^{11})}{2} \]

65. Grade
\[ c_{a,v} = \frac{c_o A_o + \frac{c}{2} (A_o + 2A_1 + 2A_2 + \ldots + A_n)}{A_o} \]

Method of triangles

67. For triangular prism - volume
\[ V = \frac{1}{3} (t_1 + t_2 + t_3) S \]

68. For triangular prism - grade
\[ c_{a,v} = \frac{c_1 + c_2 + c_3}{3} \]

69. For triangular prism - grade
\[ c_{a,v} = \frac{c_1 t_1 + c_2 t_2 + c_3 t_3}{t_1 + t_2 + t_3} \]

70. Ore tonnage
\[ Q = \frac{V}{F} \text{ or } \frac{(q_1 + q_2 + q_3)}{3} S \]
71. Metal tonnage

\[ P = Qc_{av} \text{ or } \left( \frac{q_1 c_1 + q_2 c_2 + q_3 c_3}{3} \right) S \text{ or } \left( \frac{P_1 + P_2 + P_3}{3} \right) S \]

72. Number of connecting lines

\[ N_l = 3n_1 + 2n_2 + 3, \text{ or } N_l = 3(n_1 - 1) + 2n_2 \]

73. Number of triangles

\[ N_{tr} = 2n_1 + n_2 - 2 = 2(n_1 - 1) + n_2 \]

74. The relative error for rectangular prism

\[ \Delta V = V_1 - V_2 = \frac{1}{6} (t_1 + 2t_2 + t_3 + 2t_4) S - \frac{1}{6} (2t_1 + 2t_2 + 2t_3 + t_4) S \]

or \[ \Delta V = \pm \frac{S}{6} (t_1 - t_2 + t_3 - t_4) \]

75. Condition for precise volume computations in rectangular prism

\[ t_1 + t_3 = t_2 + t_4 \]

76. General case - average thickness

\[ t_{av} = \frac{4t_1 + t_2 + t_3}{6} \]

77. Case of \( t_3 = 0 \)

\[ t_{av} = \frac{3t_3 + t_2}{6} \]

78. Case of \( t_2 \) and \( t_3 = 0 \)

\[ t_{av} = \frac{t_1}{3} \]

79. Volume for the entire body \( (s = \frac{S}{N}) \)

\[ V = \frac{1}{3} s \sum_{i=1}^{N} tk \]

80. Ore reserves for the entire body \( (s = \frac{S}{N}) \)

\[ Q = \frac{1}{3} s \sum_{i=1}^{N} t_{fk} \]
81. Metal reserves for the entire body \( s = \frac{S}{N} \)

\[
P = \frac{1}{3} s \sum_{i=1}^{N} t_{fck}
\]

82. Metal tonnage

\[
P = c_1 q_1 + c_2 q_2 + c_3 q_3 + \ldots + c_n q_n
\]

83. Square net of workings - one block

\[
V = \frac{(t_1 + t_2 + t_3 + t_4)}{4} s
\]

84. Square net of workings for the entire body \( s = \frac{S}{N} \)

\[
V = \frac{t_1 + t_2 + t_3 + \ldots + t_n}{N} S = (t_1 + t_2 + t_3 + \ldots + t_n) s
\]

85. Total average relative error

\[
M_{\Delta V} = \sqrt{m_1^2 + m_2^2 + m_3^2}
\]
APPENDIX E. - GLOSSARY

Selected terms used in this report are included in the following definitions.

**Area reserves.** - Reserves computed for a certain area.

**Arithmetic average or mean.** - Simple average of a suite of quantities, measurements, analyses, etc.; sum of a suite divided on the number of quantities.

**Block.** - A unit of mineral body delineated by various principles of interpretation of exploration data; varies with the method of computations.

**Inside perimeter.** - The portion of a mineral deposit delineated by outlying mine workings.

**Linear metal reserves.** - Metal reserves for an area unit, such as square foot (product of linear ore reserves and average grade), in tons or other weight units.

**Linear ore reserves.** - Ore reserves computed for an area unit, such as square foot or square meter, in tons or other weight units.

**Mine workings.** - All surface and underground exploration, development, and exploitation exposures of a mineral body; drilling included.

**Ore.** - A mineral substance that can be mined at present at profit to the operator or to the benefit of the nation.

**Outside perimeter.** - Portion of a mineral deposit extended beyond the outlying mine workings; delineated according to geological evidence or certain principles.

**Parameters of a mineral body.** - A series of physical and chemical constants which express this body.

**Reserves.** - Mineral material considered exploitable under existing conditions; including cost, price, technology, and local circumstances.

**Resources.** - Reserves plus potential raw material; includes marginal, submarginal, and latent categories.

**Section reserves.** - Reserves along a section--one unit of length wide.

**Segment.** - A large portion of a mineral body.

**Underground mine workings.** - Exploration and development shafts, adit, drifts, crosscuts, raises, and winzes; drilling excluded.

**Unit volume reserves.** - Reserves computed for one unit of volume, such as cubic foot, cubic yard, or cubic meter.

1Definition approved by the United Nations Educational, Scientific and Cultural Organizations.